Neutrino Masses

Contents

I. THE RENORMALIZABLE STANDARD MODEL

We defined the Standard Model (SM) as follows:

(i) The symmetry is a local

$$SU(3)_C \times SU(2)_L \times U(1)_Y, \tag{1.1}$$

that is spontaneously broken into

$$SU(3)_C \times U(1)_{\rm EM}, \tag{1.2}$$

by the VEV of a single scalar multiplet:

$$\phi(1,2)_{+1/2}. \tag{1.3}$$

(ii) There are three fermion generations, each consisting of five different representations (we omit the chirality indices L (for Q_L and L_L) and R (for U_R, D_R and E_R)):

$$Q_i(3,2)_{+1/6}, \quad U_i(3,1)_{+2/3}, \quad D_i(3,1)_{-1/3}, \quad L_i(1,2)_{-1/2}, \quad E_i(1,1)_{-1}. \tag{1.4}$$

The neutrinos are the $T_3 = +1/2$ members of the lepton doublets L_i. The Lagrangian terms that include the lepton doublets are the following:

$$\mathcal{L}_L = i\overline{L_i}\gamma_\mu \left(\partial^\mu + \frac{i}{2}gW_b^\mu\tau_b + \frac{i}{2}g'B^\mu\right) L_i - \left(Y^e_{ij}\overline{L_i}\phi E_j + \text{h.c.}\right). \tag{1.5}$$

Moving to the mass basis, we define ν_α as the $SU(2)_L$-doublet partner of the charged lepton mass eigenstate ℓ_α:

$$\mathcal{L}_\nu = i\overline{\nu_\alpha}\gamma^\mu\partial_\mu\nu_\alpha - \frac{g}{2\cos\theta_W}\overline{\nu_\alpha}\gamma^\mu Z^0_\mu\nu_\alpha - \frac{g}{\sqrt{2}}\left(\overline{\ell_\alpha}\gamma^\mu W^-_\mu\nu_\alpha + \text{h.c.}\right). \tag{1.6}$$

Thus, neutrinos have charged current and neutral current weak interactions, but no Yukawa interactions and no mass terms.

As concerns neutrino masses, the absence of right-handed neutrinos N_R (which are SM singlets $(1,1)_0$) prevents Dirac mass terms, which are of the form $m_D\overline{N_R}\nu_L$. In principle, Majorana mass terms, of the form $\frac{1}{2}m_M\overline{(\nu_L)^c}\nu_L$, are consistent with the $SU(3)_C \times U(1)_{\rm EM}$ symmetry, yet, as evident in Eq. (1.6), they do not arise. This can be understood by symmetry considerations. The renormalizable SM Lagrangian has an accidental $B - L$ symmetry. A Majorana mass term for neutrinos would violate this symmetry by two units. Thus, the symmetry prevents mass terms not only at tree level but also to all orders in perturbation theory. Moreover, since the symmetry is non-anomalous (unlike B and L separately), Majorana mass terms do not arise even at the non-perturbative level. We conclude that the renormalizable SM predicts

$$m_\nu = 0 \tag{1.7}$$

to all orders in perturbation theory, and beyond.

II. THE NON-RENORMALIZABLE STANDARD MODEL

The SM is not a full theory of Nature. It is only a low energy effective theory, valid below some scale $\Lambda(\gg \Lambda_{\rm EW})$. Then, the SM Lagrangian should be extended to include all non-renormalizable terms, suppressed by powers of Λ:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}O_{d=5} + \frac{1}{\Lambda^2}O_{d=6} + \cdots, \tag{2.1}$$

where $O_{d=n}$ represents an operator that is a product of SM fields, a singlet under the SM gauge group, of overall dimension n in the fields. The important point is that since $B - L$ is an accidental symmetry of the SM, there is no reason to expect that it will be respected by the non-renormalizable terms.

Indeed, already at $d = 5$, $B - L$ is violated:

$$\mathcal{L}_L = \mathcal{L}_L^{\rm SM} + \frac{Z^\nu_{ij}}{\Lambda}\phi\phi L_i L_j + \mathcal{O}\left(\frac{1}{\Lambda^2}\right). \tag{2.2}$$

The inclusion of these terms leads to significant changes in the phenomenology of the lepton sector. The modifications that can be presently tested by experiments can be presented by re-writing the neutrino-related terms in the mass basis. The Lagrangian of Eq. (1.6) is modified as follows:

$$\mathcal{L}_\nu = i\overline{\nu_i}\gamma^\mu\partial_\mu\nu_i - \frac{g}{2\cos\theta_W}\overline{\nu_i}\gamma^\mu Z^0_\mu\nu_i - \frac{g}{\sqrt{2}}\left(\overline{\ell_i}\gamma^\mu U_{ij}W^-_\mu\nu_j + \text{h.c.}\right) + m_i\nu_i\nu_i + \cdots. \qquad (2.3)$$

where $m_{1,2,3}$ are real, and U is unitary. Here $\nu_{1,2,3}$ are the neutrino mass eigenstates.

The significant modifications are then the following:

- The neutrinos have Majorana masses.
- The leptonic charged current interactions are not universal. They involve a mixing matrix.

How many physical parameters are involved in the lepton sector? The Lagrangian of Eq. (2.2) involves the 3×3 matrix Y^e (9 real and 9 imaginary parameters) and the symmetric 3×3 matrix Z^ν (6 real and 6 imaginary parameters). The kinetic and gauge terms have a $U(3)_L \times U(3)_E$ accidental global symmetry, that is completely broken by the Y^e and Z^ν terms. Thus, the number of physical parameters is $(15_R + 15_I) - 2\times(3_R + 6_I) = 9_R + 3_I$. Six of the real parameters are the three charged lepton masses $m_{e,\mu,\tau}$ and the three neutrino masses m_i. We conclude that the 3×3 unitary matrix U depends on three real mixing angles and three phases.

Why does the lepton mixing matrix U depend on three phases, while the quark mixing matrix V depends on only a single phase? The reason for this difference lies in the fact that the Lagrangian of Eq. (2.2) leads to Majorana masses for neutrinos. Consequently, there is no freedom in changing the mass basis by redefining the neutrino phases, as such redefinition will introduce phases into the neutrino mass terms. While redefinitions of the six quark fields allowed us to remove five non-physical phases from V, redefinitions of the three charged lepton fields allows us to remove only three non-physical phases from U. The two additional physical phases in U are called "Majorana phases", since they appear as a result of the Majorana nature of neutrinos. They affect lepton number violating processes.

In the interaction basis, the neutrino mass matrix has the form

$$M_\nu = Z^\nu \frac{\langle\phi\rangle^2}{\Lambda}. \qquad (2.4)$$

Since, by definition, $\langle\phi\rangle \ll \Lambda$, the model predicts not only that the neutrinos are massive, $m_\nu \neq 0$, but also that they are, in general, much lighter than the charged fermions:

$$m_\nu \sim \frac{\langle\phi\rangle^2}{\Lambda} \ll \langle\phi\rangle \sim m_{\ell,q}. \qquad (2.5)$$

The fact that experiments indeed find that the neutrinos are lighter by at least six orders of magnitude than the lightest charged fermion (the electron) makes the notion that neutrino masses are generated by $d = 5$ terms very plausible.

Clearly, the SM cannot be a valid theory above the Planck scale. Since $\langle\phi\rangle^2/M_{\rm Pl} \sim 10^{-5}$ eV, we expect $m_\nu \gtrsim 10^{-5}$ eV. If the relevant scale is, for example, the Grand Unification scale, then we expect $m_\nu \sim 10^{-2}$ eV.

III. THE SEE-SAW MECHANISM

Let us add to the SM gauge-singlet fermions,

$$N_i(1,1)_0. \tag{3.1}$$

Then new N_i-dependent terms appear in the (renormalizable) Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_N, \tag{3.2}$$

where $\mathcal{L}_N$ contains Yukawa and Majorana mass terms:

$$\mathcal{L}_N = i\overline{N_i}\gamma_\mu\partial^\mu N_i - \left(Y^\nu \overline{L_i}\tilde{\phi}N_j + M^N_{ij}N_iN_j + {\rm h.c.}\right). \tag{3.3}$$

Let us count the number of physical parameters in this model. For the sake of definiteness, we assume that there are three N_i fields. The leptonic sector depends on Y^e $(9_R + 9_I)$, Y^ν (9_R+9_I), and M^N (6_R+6_I). When we take all of these parameters to zero, we gain a global $U(3)_L\times U(3)_E\times U(3)_N$, which is completely broken when all Yukawa and mass parameters are switched on. The number of physical parameters is then $(24_R+24_I)-3(3_R+6_I) = 15_R+6_I$. Nine of the real parameters are the three masses of charged leptons, three masses of light neutrinos and three masses of heavy neutrinos.

Of particular interest to us is the 6×6 neutrino mass matrix, that can be decomposed into four 3×3 blocks as follows:

$$M_\nu = \begin{pmatrix} 0 & Y^\nu\langle\phi\rangle \\ Y^{\nu T}\langle\phi\rangle & M^N \end{pmatrix}. \tag{3.4}$$

Let us assume that the eigenvalues of M^N are much larger than the electroweak breaking scale, $M_{1,2,3} \gg \langle\phi\rangle$. Then, we can bring M_ν to a block-diagonal form, with $M_{\rm heavy} \simeq M^N$ and

$$M_{\rm light} \simeq \langle\phi\rangle^2 Y^\nu (M^N)^{-1} Y^{\nu T}. \tag{3.5}$$

The scale of the light neutrino masses, $\langle\phi\rangle^2/m_N$, where m_N is the scale of the heavy neutrino masses, is much lower than the scale of the charged lepton masses, $\langle\phi\rangle$. This situation should remind you of our discussion of the neutrino masses when we include dimension five terms.

Indeed, integrating out the heavy gauge-singlet fermions generates the non-renormalizable terms of Eq. (2.2). Within this model, the scale to which the dimension-five terms are inversely proportional acquires a concrete interpretation,

$$Z^{\nu}_{ij}/\Lambda = (Y^{\nu}(M^{N})^{-1}Y^{\nu T})_{ij}. \tag{3.6}$$

If, for example, the heavy neutrino are degenerate with a common mass m_N, then we can take $\Lambda = m_N$ and $Z^{\nu}_{ij} = Y^{\nu}_{ik}Y^{\nu}_{jk}$.

Clearly, the heavier the (approximately) gauge-singlet neutrinos are, the lighter the (approximately) $SU(2)_L$-doublet neutrinos are. For this reason, the mechanism that generates light neutrino masses via their Yukawa couplings to heavy neutrinos is called "the see-saw mechanism." It arises naturally in various extensions of the SM, such as $SO(10)$ grand unified theory (GUT), and left-right-symmetric (LRS) models.

IV. VACUUM OSCILLATIONS

In experiments, neutrinos are produced and detected by charged current weak interactions. Thus, the states that are relevant to production and detection are the interaction eigenstates

$$\nu_e, \quad \nu_\mu, \quad \nu_\tau. \tag{4.1}$$

On the other hand, the eigenstates of free propagation in space-time are the mass eigenstates,

$$\nu_1, \quad \nu_2, \quad \nu_3. \tag{4.2}$$

In general, the interaction eigenstates are different from the mass eigenstates:

$$|\nu_\alpha\rangle = U^*_{\alpha i}|\nu_i\rangle, \quad (\alpha = e, \mu, \tau,\ i = 1, 2, 3). \tag{4.3}$$

Consequently, flavor is not conserved during propagation in space-time and, in general, we may produce ν_α but detect $\nu_{\beta\neq\alpha}$.

The probability $P_{\alpha\beta}$ of producing neutrinos of flavor α and detecting neutrinos of flavor β is calculable in terms of

- The neutrino energy E;
- The distance between source and detector L;
- The mass squared difference $\Delta m^2_{ij} \equiv m^2_i - m^2_j$
  ($P_{\alpha\beta}$ is independent of the absolute mass scale);
- The mixing matrix U parameters (mixing angles and phase)
  ($P_{\alpha\beta}$ is independent of the Majorana phases).

Starting from Eq. (4.3), we can write the expression for the time evolved $|\nu_\alpha(t)\rangle$ (where $|\nu_\alpha(0)\rangle = |\nu_\alpha\rangle$):

$$|\nu_\alpha(t)\rangle = \sum_i U^*_{\alpha i}|\nu_i(t)\rangle, \tag{4.4}$$

where

$$|\nu_i(t)\rangle = e^{-iE_i t}|\nu_i(0)\rangle. \tag{4.5}$$

Thus, the probability of a state that is produced as ν_α to be detected as ν_β is given by

$$\begin{aligned} P_{\alpha\beta} &= |\langle\nu_\beta|\nu_\alpha(t)\rangle|^2 \qquad (4.6)\\ &= |\langle\nu_\beta|\nu_i\rangle\langle\nu_i|\nu_\alpha(t)\rangle|^2 \\ &= \delta_{\alpha\beta} - 4\sum_{j>i}\mathcal{R}e\left(U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right)\sin^2\left(\frac{\Delta m^2_{ij}L}{4E}\right) \\ &\quad +2\sum_{j>i}\mathcal{I}m\left(U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right)\sin\left(\frac{\Delta m^2_{ij}L}{2E}\right). \end{aligned}$$

If we apply this calculation to the generation case, where there is a single mixing angle (and no relevant phase) and a single mass-squared difference,

$$\begin{aligned} U &= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \\ \Delta m^2 &= m_2^2 - m_1^2, \qquad (4.7) \\ & \qquad (4.8) \end{aligned}$$

we obtain, for $\alpha \neq \beta$,

$$P_{\alpha\beta} = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right). \tag{4.9}$$

This expression depends on two parameters that are related to the experimental design, E and L, and two that are parameters of the Lagrangian, Δm^2 and θ. To be sensitive to the theoretical parameters, one has to design the experiment appropriately:

$$\Delta m^2 L/E \begin{cases} \ll 1 & P_{\alpha\beta} \to 0 \\ \sim 1 & P_{\alpha\beta} \text{ sensitive to } \Delta m^2, \theta \\ \gg 1 & P_{\alpha\beta} \to \frac{1}{2}\sin^2 2\theta. \end{cases} \tag{4.10}$$

We learn that to allow observation of neutrino oscillations, Nature needs to provide $\sin^2 2\theta$ that is not too small. Furthermore, to probe small Δm^2 values, we need experiments with large L/E. Indeed, given natural sources as well as reactors, we can probe a rather large range of Δm^2; see the list in Table I.

TABLE I: Neutrino vacuum oscillation experiments

Source	E[MeV]	L[km]		Δm^2[eV^2]
Solar (VO)	1	10^8	$\Longrightarrow$	$10^{-11} - 10^{-9}$
Reactor	1	10^2	$\Longrightarrow$	$10^{-5} - 10^{-3}$
Atmospheric	10^3	10^{1-4}	$\Longrightarrow$	$10^{-5} - 1$

V. THE MSW EFFECT

The Mikheev-Smirnov-Wolfenstein (MSW) effect provides yet another way to probe neutrino mixing and masses. Consider the two neutrino. In vacuum, in the mass basis (ν_1, ν_2), the Hamiltonian can be written as

$$\mathcal{H} = p + \begin{pmatrix} \frac{m_1^2}{2E} & \\ & \frac{m_2^2}{2E} \end{pmatrix}. \tag{5.1}$$

In the interaction basis (ν_e, ν_a), where ν_a is a combination of ν_μ and ν_τ, we have

$$\mathcal{H} = p + \frac{m_1^2 + m_2^2}{4E} + \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}. \tag{5.2}$$

In matter (that is, in an (e, p, n) plasma), in the interaction basis,

$$\mathcal{H} = p + V_a + \frac{m_1^2 + m_2^2}{4E} + \begin{pmatrix} (V_e - V_a) - \frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}. \tag{5.3}$$

All active neutrinos have the same (universal) neutral current interactions. In contrast, in a plasma that has electrons but neither muons nor twu-leptons, only ν_e has charged current interactions with matter:

$$V_e - V_a = \sqrt{2} G_F n_e. \tag{5.4}$$

Thus, omitting the part in the Hamiltonian that is proportional to the unit matrix in flavor space (which plays no role in the oscillations), we have

$$\mathcal{H} \sim \begin{pmatrix} \sqrt{2} G_F n_e - \frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}. \tag{5.5}$$

We learn that the mixing angle that relates the flavor eigenstates (ν_e, ν_a) to the mass eigenstates in matter (ν_1^m, ν_2^m) depends on the matter density:

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2\sqrt{2} G_F n_e E}. \tag{5.6}$$

For example, in case of very large electron density, $\sqrt{2} G_F n_e \gg \frac{\Delta m^2}{2E}$, we have $\theta_m \simeq \pi/2$, which means that ν_e is very close to the heavier mass eigenstate ν_2^m.

Things become even more complicated for a neutrino propagating in a varying density $n_e(x)$. The mixing angle is then changing, $\theta_m = \theta_m(n_e(x))$:

$$\tan 2\theta_m(x) = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2\sqrt{2} G_F n_e(x) E}. \tag{5.7}$$

In particular, as $n_e(x)$ decreases, so does θ_m. Defining

$$n_e^R = \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2} G_F E}, \tag{5.8}$$

we have

$$n_e \begin{cases} \gg n_e^R & \theta_m \approx \pi/2, \\ = n_e^R & \theta_m = \pi/4, \\ = 0 & \theta_m = \theta. \end{cases} \tag{5.9}$$

We conclude that ν_2^m propagating along a decreasing n_e is mostly ν_e for n_e above n_e^R and mostly ν_a for n_e below n_e^R.

The propagation in varying density allows yet another interesting effect, and that is $\nu_1^m \leftrightarrow \nu_2^m$ transitions. The source of this effect is the fact that $e^{-iH(t)t} \neq e^{-i\int H(t')dt'}$, which means that the instantenous mass eugenstates are not the eigenstates of time evolution. However, for *slowly* varying density, $\dot{H}t \ll H$, we have $e^{-i\int H(t')dt'} = e^{-(iHt+\dot{H}t^2+\cdots)} \approx e^{-iH(t)t}$, and the $\nu_1^m \leftrightarrow \nu_2^m$ transitions can be neglected. The condition for neglecting these transition is known as *the adiabatic condition*:

$$\frac{1}{n}\frac{dn}{dx} \ll \frac{\Delta m^2}{E}\frac{\sin^2 2\theta}{\cos 2\theta}. \tag{5.10}$$

We now describe the characteristics of ν_e production and propagation in the Sun. The electron density in the Sun can be parameterized as $n_e(x) \approx 2n_0 \exp(-x/r_0)$, where the relevant parameters are given in Table II. Consider the case where $n_e^{\rm prod} \gg n_e^R$. Then, according to Eq. (5.9), we have at the production point $\nu = \nu_2^m$ ($\theta_m = \pi/2$). Further assume that the propagation is adiabatic at $n_e \sim n_e^R$ (Eq. (5.10) is fulfilled at that point). Then, at the resonance point we still have $\nu = \nu_2^m$ ($\theta_m = \pi/4$). Finally, as the neutrino arrives to the surface of the Sun, it is still $\nu = \nu_2^m$ but now, according to Eq. (5.9), we have $\theta_m = \theta$, and the neutrino is simply in the heavy mass eigenstate. Being a mass eigenstate, it does not oscillate along its propagation to Earth. We conclude that for solar ν_e's with energy in the range

$$\Delta m^2 G_F n_e^{\rm prod} \ll E \ll \frac{\Delta m^2}{\frac{1}{n}\frac{dn}{dx}}\frac{\sin^2 2\theta}{\cos 2\theta}, \tag{5.11}$$

the probabilty of being detected as ν_e is given by

$$P_{ee}^{\rm MSW} = |\langle \nu_e | \nu_2 \rangle|^2 = \sin^2\theta. \tag{5.12}$$

TABLE II: Solar neutrino (MSW) experiments

Source	$n_0[\text{cm}^{-3}]$	$r_0[\text{cm}]$		$\Delta m^2[\text{eV}^2]$
Solar (MSW)	6×10^{-25}	7×10^9	$\Longrightarrow$	$10^{-9} - 10^{-5}$

It is highly sensitive to θ and provides the only way to probe small mixing angles. Indeed, for $\Delta m^2 \sim 10^{-4}$ eV, solar neutrinos would have allowed probing a mixing angle as small as $\sin^2\theta \sim 10^{-4}$.

On the other hand, for solar ν_e's with energy in the range

$$E \ll \Delta m^2 \cos 2\theta G_F n_e^{\text{prod}}, \tag{5.13}$$

namely $n_e^{\text{prod}} \ll n_e^R$, the produced state is $\nu = \sin\theta\nu_2^m + \cos\theta\nu_1^m$. Approaching the surface of the Sun, $\nu = \sin\theta\nu_2 + \cos\theta\nu_1 = \nu_e$ and $P_{ee}(R_\odot) = 1$. Along the propagation to Earth, the neutrino is subject to vacuum oscillations, with the final result (see Eq. (4.10))

$$P_{ee}^{\text{VO}} = 1 - \frac{1}{2}\sin^2 2\theta. \tag{5.14}$$

Note that $P_{ee}^{\text{MSW}} < \frac{1}{2}$ is possible, while $P_{ee}^{\text{VO}} > \frac{1}{2}$. For solar neutrinos, the transition between those subject to the MSW effect, Eq. (5.12), and those subject to vacuum oscillations, Eq. (5.14), occurs at $E \sim$MeV. The relevant parameters are given in Table II.

Examining Tables I and II, we conclude that, if $\theta \not\ll 1$, neutrino masses in the entire theoretically interesting range, 10^{-11} eV$^2 \ll \Delta m^2 \ll$ eV^2 could be discovered. For $10^{-2} \lesssim \theta \ll 1$, neutrino masses could still be discovered via the adiabatic MSW effect for $\Delta m^2 \sim 10^{-5}$ eV^2.

VI. EXPERIMENTAL RESULTS

Neutrino flavor transitions have been observed for solar, atmospheric, reactor and accelerator neutrinos. Five neutrino flavor parameters – two mass-squared differences and the three mixing angles – have been measured:

$$\begin{aligned}
\Delta m_{21}^2 &= (7.5 \pm 0.3) \times 10^{-5}\ eV^2, \\
|\Delta m_{32}^2| &= (2.4 \pm 0.1) \times 10^{-3}\ eV^2, \\
\sin^2\theta_{12} &= 0.31 \pm 0.02, \\
\sin^2\theta_{23} &= 0.40 \pm 0.03, \\
\sin^2\theta_{13} &= 0.025 \pm 0.003.
\end{aligned} \tag{6.1}$$

Note that the convention for naming the neutrino mass eigenstates is as follows. The two states separated by the smaller mass-squared difference are called ν_2 and ν_1, with ν_2 the heavier among the two. The mass eigenstate separated from these two by the larger mass-squared difference is called ν_3. It could be heavier ("normal hierarchy") or lighter ("inverted hierarchy") than the other two.

Also note that the following physical questions are still open:

- The absolute mass scale of the neutrino is still unknown. On one extreme, they could be quasi-degenerate and as heavy as parts of eV. On the other extreme, they could be hierarchical, with the lightest possibly massless.
- It is not known whether the spectrum has normal or inverted hierarchy.
- None of the phases has been measured.
- The question of whether the neutrinos are Dirac or Majorana particles still has no experimental answer.

VII. LESSONS FROM NEUTRINO MASSES

As should be obvious by now, neutrino masses provide experimental evidence that *the Standard Model is not a complete theory of nature.* Most likely, the SM is a good low energy effective theory. Lepton number is broken at some high energy scale, generating neutrino masses via the dimension-five terms of Eq. (2.2). Assuming that, at least some of the dimensionless Z^ν_{ij} couplings are of order one, we can estimate the scale at which lepton number is broken via

$$\Lambda_L \sim \frac{\langle\phi\rangle^2}{m_\nu}. \tag{7.1}$$

Alternatively, taking $Z^\nu_{ij} \lesssim 1$, Eq. (7.1) turns into an upper bound on the scale. Since at least one of the neutrinos has a mass

$$m_\nu \geq \sqrt{\Delta m^2_{32}} \simeq 0.05 \text{ eV}, \tag{7.2}$$

we conclude that the SM breaks at a scale

$$\Lambda_L \lesssim 10^{15} \text{ GeV}. \tag{7.3}$$

This proves that the SM cannot be valid up to the Planck scale. The upper bound is also intriguingly close to the GUT scale.

Let us expand a little about the issue of grand unified theories. The main motivations to consider these theories are gauge coupling unification, gauge multiplet unification, and (for the third generation) flavor unification. The main reasons to be suspicious about GUTs are the non-observation of proton decay, the doublet-triplet splitting, the flavor splitting (for the first two generations), and the non-observation of supersymmetry. The neutrino parameters strengthen the case for grand unification:

- $m_\nu \neq 0$: In SO(10) theories, singlet fermions exist, and their Yukawa couplings are related to those of the up quarks. Hence, these models predict that $m_\nu \neq 0$.

- $m_\nu \sim 0.05$ eV: In SO(10) models, $M_\nu^{\rm Dirac} = M_u$, and $\Lambda_{SO(10)} \sim 10^{16}$ GeV. Hence we obtain $m_{\nu_3} \sim \frac{m_t^2}{\Lambda_{SO(10)}} \sim 10^{-3}$ eV.

- In SU(5) models, $M_\ell = M_d^T$. Consequently, we expect $|U_{\mu 3}| \sim \frac{m_s/m_b}{|V_{cb}|} = \mathcal{O}(1)$.

VIII. LEPTOGENESIS

The leptonic part of the Lagrangian, when singlet fermions N_i are added, is given in Eq. (3.3). The addition of the Y^ν and M terms, involving the N_i's, is motivated by the seesaw mechanism for light neutrino masses. When the heavy N_i's are integrated out, an effective mass matrix for the light neutrinos is generated, Eq. (3.5). It can be rewritten as follows (for simplicity, we replace Y^ν with λ in this section):

$$\begin{aligned} \mathcal{L}_{m^\nu} &= \frac{1}{2}\overline{\nu^c}_\alpha m^\nu_{\alpha\beta}\nu_\beta + \text{h.c.}, \\ m^\nu_{\alpha\beta} &= \lambda_{\alpha k} M_k^{-1} \lambda_{\beta k} v^2. \end{aligned} \tag{8.1}$$

The addition of these terms also implies, however, that the physics of the singlet fermions is likely to play a role in dynamically generating a lepton asymmetry in the Universe. The reason that leptogenesis is qualitatively almost unavoidable once the seesaw mechanism is invoked is that the Sakharov conditions are (likely to be) fulfilled:

1. Lepton number violation: The Lagrangian terms (3.3) violate L because lepton number cannot be consistently assigned to N_1 in the presence of λ and M. If $L(N_1) = 1$, then $\lambda_{\alpha 1}$ respects L but M_1 violates it by two units. If $L(N_1) = 0$, then M_1 respects L but $\lambda_{\alpha 1}$ violates it by one unit. (Remember that the fact that the SM interactions violate $B + L$ implies that the requirement for baryogenesis from new physics is $B - L$ violation and not necessarily B violation.)

2. CP violation: Since there are irremovable phases in λ (once Y^e and M are chosen to be real), the Lagrangian terms (3.3) provide new sources of CP violation.

3. Departure from thermal equilibrium: The interactions of the N_i's are only of the Yukawa type. If the λ couplings are small enough, these interactions can be slower than the expansion rate of the Universe, in which case the singlet fermions will decay out of equilibrium.

Thus, in the presence of the seesaw terms, leptogenesis is *qualitatively* almost unavoidable, and the question of whether it can successfully explain the observed baryon asymmetry is a *quantitative* one.

We consider leptogenesis via the decays of N_1, the lightest of several (at least two) singlet neutrinos N_i. When the decay is into a single flavor α, $N_1 \to L_\alpha \phi$ or $\bar{L}_\alpha \phi^\dagger$, the baryon asymmetry can be written as follows:

$$Y_{\Delta B} \simeq \left(\frac{135\zeta(3)}{4\pi^4 g_*}\right) \times C_{\rm sphal} \times \eta \times \epsilon. \tag{8.2}$$

The first factor is the equilibrium N_1 number density divided by the entropy density at $T \gg M_1$. It is of $\mathcal{O}(4 \times 10^{-3})$ when the number of relativistic degrees of freedom g_* is taken as in the SM, $g_*^{\rm SM} = 106.75$. The other three factors on the right hand side of Eq. (8.2) represent the following physics aspects:

1. ϵ is the CP asymmetry in N_1 decays. For every $1/\epsilon$ N_1 decays, there is one more L than there are $\bar{L}$'s.

2. η is the efficiency factor. Inverse decays, other "washout" processes, and inefficiency in N_1 production, reduce the asymmetry by $0 < \eta < 1$. In particular, $\eta = 0$ is the limit of N_1 in perfect equilibrium, so no asymmetry is generated.

3. $C_{\rm sphal}$ describes further dilution of the asymmetry due to fast processes which redistribute the asymmetry that was produced in lepton doublets among other particle species. These include gauge, Yukawa, and $B + L$ violating non-perturbative effects.

These three factors can be calculated, with ϵ and η depending on the Lagrangian parameters. The final result can be written (with some simplifying assumptions) as

$$Y_{\Delta B} \sim 10^{-3}\, \frac{10^{-3}\ {\rm eV}}{\tilde{m}}\, \epsilon, \tag{8.3}$$

where ($x_j \equiv M_j^2/M_1^2$)

$$\epsilon = \frac{1}{8\pi} \frac{1}{(\lambda^\dagger\lambda)_{11}} \sum_j \mathcal{I}m \left\{ \left[(\lambda^\dagger\lambda)_{1j}\right]^2 \right\} \sqrt{x_j} \left[\frac{1}{1-x_j} + 1 - (1+x_j) \ln\left(\frac{1+x_j}{x_j}\right) \right], \tag{8.4}$$

and

$$\tilde{m} = \frac{(\lambda^\dagger\lambda)_{11} v^2}{M_1}. \tag{8.5}$$

The plausible range for $\tilde{m}$ is the one suggested by the range of hierarchical light neutrino masses, $10^{-3} - 10^{-1}$ eV, so we expect a rather mild washout effect, $\eta \gtrsim 0.01$. Then, to account for $Y_{\Delta B} \sim 10^{-10}$, we need $|\epsilon| \gtrsim 10^{-5} - 10^{-6}$. Using Eq. (8.4), we learn that this condition roughly implies, for the seesaw parameters,

$$\frac{M_1}{M_2} \frac{\mathcal{I}m[(\lambda^\dagger \lambda)^2_{12}]}{(\lambda^\dagger \lambda)_{11}} \gtrsim 10^{-4} - 10^{-5}, \tag{8.6}$$

which is quite natural. More concretely, taking as rough estimate $\lambda^2 v^2 / M_1 \sim 10^{-2}$ eV, then $\lambda \gtrsim 10^{-2}$ is very plausible for $M_1 \gtrsim 10^{11}$ GeV.

We can thus conclude that leptogenesis is attractive not only because all the required features are qualitatively present, but also because the quantitative constraints are plausibly satisfied. In particular, $\tilde{m} \sim 0.01$ eV, as suggested by the light neutrino masses, is optimal for thermal leptogenesis as it leads to effective production of N_1's in the early Universe and only mild washout effects. Furthermore, the required CP asymmetry can be achieved in large parts of the seesaw parameter space.