Quantum Mechanics II Problem Set #3 due 17/12/97

I. Groups and Representations

1. If x is an element of a group G, prove that the inverse element x^{-1} is unique.

2. Find the multiplication law for a group with three elements and prove that it is unique.

3. Show that the representation (1.2) of the permutation group is reducible. (Hint: find a vector which is an eigenvector of all the D's, then take it from there.)

II. Lie Groups and Lie Algebras

1. Find all components of the matrix $\exp(i\alpha A)$, where

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

2. If $[C, B] = B$, calculate $\exp(i\alpha C) B \exp(-i\alpha C)$.

3. Carry out the expansion of δ_c in (2.4) to third order in α and β.

4. Prove Schur's Lemma, which states that if a matrix M commutes with all the matrix generators of an irreducible representation of a Lie algebra, M is proportional to the identity matrix. (Hint: assume M is hermitian and diagonalize it.)