Quantum Mechanics II Problem Set #7 due 21/1/98

1 Lorentz Transformations

1. Show that any *Lorentz Transformation* $\boldsymbol{\Lambda}$ can be written as

$$\boldsymbol{L} \times \text{space inversion} \quad \text{for } \boldsymbol{\Lambda} \in L_-^\uparrow \tag{1}$$
$$\boldsymbol{L} \times \text{time inversion} \quad \text{for } \boldsymbol{\Lambda} \in L_-^\downarrow \tag{2}$$
$$\boldsymbol{L} \times \text{space inversion} \times \text{time inversion} \quad \text{for } \boldsymbol{\Lambda} \in L_+^\downarrow, \tag{3}$$

   where $\boldsymbol{L} \in L_+^\uparrow$.

2. Show that a restricted Lorentz transformation ($\boldsymbol{\Lambda} \in L_+^\uparrow$) can be uniquely written as the product of a boost and a rotation.

2 One Particle State

1. Suppose that observer $\mathcal{O}$ sees a W-boson (spin one and mass $m_W \neq 0$) with momentum $\mathbf{p}$ in the x-direction and spin z-component σ. A second observer $\mathcal{O}'$ is rotated in respect to $\mathcal{O}$ by $+90$ degrees in the $z-x$ plane. How does O' describe the W-state?

2. A third observer $\mathcal{O}''$ moves relative $\mathcal{O}$ with velocity $\mathbf{v}$ in the z-direction. How does $\mathcal{O}''$ describe the W-state?

   Hint: Consider first infinitesimal transformations.

3 Casimir Operators of the Lorentz Group

1. Show that the operators $P_\mu P^\mu$ and $W_\mu W^\mu$ commute with all Lorentz transformation operators $U(\boldsymbol{\Lambda}, a)$, where $W_\mu \equiv \epsilon_{\mu\nu\rho\lambda} J^{\nu\rho} P^\lambda$.

2. Compute the eigenvalues of the two Casimir operators.