Quantum Mechanics II   Problem Set #5   due 31/12/97

I. Path Integrals

1. Prove the operator relation $[\hat{p}, \hat{q}] = -i\hbar$ in the path integral formulation. Hint: define $[\hat{p}, \hat{q}]$ as $\lim_{\Delta t \to 0}(\hat{p}(t + \Delta t)\hat{q}(t) - \hat{q}(t)\hat{p}(t + \Delta t))$ and calculate its matrix element in the path integral formulation.

II. Direct Product Representations

1. Suppose that $(\sigma_a)_{ij}$ and $(\eta_a)_{xy}$ are Pauli matrices in two different two dimensional spaces. In the four dimensional tensor product space, define the basis

$$\begin{aligned} |1\rangle &= |i=1\rangle\, |x=1\rangle\,, \quad & |2\rangle &= |i=1\rangle\, |x=2\rangle\,, \\ |3\rangle &= |i=2\rangle\, |x=1\rangle\,, \quad & |4\rangle &= |i=1\rangle\, |x=1\rangle\,. \end{aligned}$$

Write out the matrix elements of $\sigma_2 \otimes \eta_1$ in this basis.

2. We will often abbreviate the tensor product notation by leaving out the indices and the identity matrices. This makes for very compact equations, but you must keep your wits about you to stay in the right space. In the example of problem (II.1), we could write:

$$\begin{aligned} (\sigma_a)_{ij}(\eta_b)_{xy} &\text{ as } \sigma_a\eta_b; \\ (\sigma_a)_{ij}\delta_{xy} &\text{ as } \sigma_a; \\ \delta_{ij}(\eta_a)_{xy} &\text{ as } \eta_a; \\ \delta_{ij}\delta_{xy} &\text{ as } 1. \end{aligned}$$

To get some practice with this notation, calculate

$$\begin{aligned} &(a)\ [\sigma_a, \sigma_b\eta_c], \\ &(b)\ \mathrm{tr}(\sigma_a\{\eta_b, \sigma_c\eta_d\}), \\ &(c)\ [\sigma_1\eta_1, \sigma_2\eta_2]. \end{aligned}$$