Elementary Particles II   Problem Set #8   due 30/1/03

Neutrino Oscillations

1. In deriving the oscillation probabilities we assumed that the 3-momentum of all constituent eigenstates are equal. Show that, when we use the relativistic approximation, the same probabilities are obtained if instead we assume that the energies of all constituent eigenstates are equal.

2. Derive the oscillation probability (as a function of the mixing matrix elements U_{ij} and the mass-squared differences Δm_{ij}^2) for N generations.

3. Consider N generations of neutrinos. If the energies are such that the oscillatory terms average to zero, show that

$$P(\nu_e \to \nu_e) \geq \frac{1}{N}.$$

4. For two neutrino generations of Majorana neutrinos, there is a physical CP violating phase in the mixing matrix. Show that this phase does not appear in the oscillation probabilities.

5. Write down the equation of motion of a neutrino, using the free Lagrangian augmented with a term

$$\mathcal{L}_{\text{eff}} = -\sqrt{2} G_F n_e \overline{\nu_{eL}} \gamma_0 \nu_{eL}.$$

Show that the effect of this term is to change the effective energy of the neutrino:

$$E = \sqrt{|\vec{p}|^2 + m^2} + \sqrt{2} G_F n_e.$$