Elementary Particles II   Problem Set #7   due 23/1/03

The Triplet Majoron Model

Define $\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W}$. Consider an arbitrary number of scalar multiplets, each characterized by its weak isospin T_i and hypercharge Y_i and carrying a VEV v_i. Then the tree level value of ρ is

$$\rho_0 = \frac{\sum_i |v_i|^2 [T_i(T_i+1) - Y_i^2]}{2\sum_i |v_i|^2 Y_i^2}.$$

1. Given that experimental constraints give

$$\rho_0 = 1.0012^{+0.0023}_{-0.0014},$$

find the maximal allowed value of the VEV of $\Delta_L(1,3)_1$ (an $SU(2)_\mathrm{L}$-triplet of hypercharge $Y=1$).

2. Define the components of the neutral member of Δ_L according to

$$\Delta_L^0 = \sqrt{\frac{1}{2}}(v_G + R_G + iI_G),$$

where $v_G = \sqrt{2}\langle\Delta_L^0\rangle$. If v_G spontaneously breaks lepton number symmetry, then I_G is the corresponding massless Goldstone boson, that is the *majoron*. Argue that in this case,

$$m^2(R_G) = \mathcal{O}(v_G^2).$$

3. Following the results of (2) and (3), it should be clear that I_G and, to a good approximation, R_G are massless (compared to m_Z^2).

(a) Calculate in this limit the decay rate $\Gamma(Z \to I_G R_G)$.

(b) Express this partial width in units of Γ_ν, the Z decay width into a single neutrino generation. Asses the viability of the model in view of the experimental value of the invisible width:

$$\frac{\Gamma_\mathrm{inv}}{\Gamma_\nu} = 2.984 \pm 0.008.$$