Elementary Particles II   Problem Set #3   due 5/12/02

I. Two Generation Mixing

1. Imagine that none of the third generation particles have been observed. Consider the direct measurements of the mixing elements in the 2×2 mixing matrix $V_{\rm C}$ for two quark generations. Are these measurements consistent with unitarity? (In other words, could we learn from these measurements about the existence of a third generation?) Knowing the values of $|V_{ub}|$ and $|V_{cb}|$, what accuracy is needed in the measurements of the $V_{\rm C}$ matrix elements to observe violation of unitarity?

2. Consider $\Delta m_K = 2|M_{12}(K)|$ as an indirect measurement of the mixing parameters in $V_{\rm C}$. Is this measurement consistent with the predictions from direct measurements and $V_{\rm C}$-unitarity? (In other words, could we learn from Δm_K about the existence of a third generation?) Take $S_0(x_c) = 2.62 \times 10^{-4}$ and $B_K = 0.85 \pm 0.15$.

II. Three Flavors and FCNC

Consider a model with only three quark flavors: u, d and s. The s_L fields is then an $SU(2)_{\rm L}$ singlet:

$$\begin{array}{lll} Q_L(3,2)_{+1/6}, & s_L(3,1)_{-1/3}, & \\ u_R(3,1)_{+2/3}, & d_R(3,1)_{-1/3}, & s_R(3,1)_{-1/3}. \end{array}$$

3. Write down the charged current weak interactions of the quark fields in the interaction basis and in the mass basis. Choose a parameterization of the mixing matrix with the minimal number of parameters.

4. Derive the neutral current weak interactions of the left-handed down quarks. In particular, show that the Z has flavor changing couplings and find their strength in terms of the charged current mixing parameters.

5. Estimate the ratio $\Gamma(K^0 \to \mu^+\mu^-)/\Gamma(K^+ \to \mu^+\nu_\mu)$ in this model. Compare to the experimental value (that can be found in the PDG).