INTRODUCTION

I. THE STANDARD MODEL

A model of the basic interactions between elementary particles is defined by the following three ingredients:

1. The symmetries of the Lagrangian;
2. The representations of fermions and scalars;
3. The pattern of spontaneous symmetry breaking.

The Standard Model (SM) is defined as follows:

1. The gauge symmetry is
$$G_{\rm SM} = SU(3)_{\rm C} \times SU(2)_{\rm L} \times U(1)_{\rm Y}. \tag{1.1}$$

2. There are three fermion generations, each consisting of five representations:
$$Q^I_{Li}(3,2)_{+1/6}, \quad u^I_{Ri}(3,1)_{+2/3}, \quad d^I_{Ri}(3,1)_{-1/3}, \quad L^I_{Li}(1,2)_{-1/2}, \quad \ell^I_{Ri}(1,1)_{-1}. \tag{1.2}$$
Our notations mean that, for example, the left-handed quarks, Q^I_L, are in a triplet (3) of the $SU(3)_{\rm C}$ group, a doublet (2) of $SU(2)_{\rm L}$ and carry hypercharge $Y = Q_{\rm EM} - T_3 = +1/6$. The index I denotes interaction eigenstates. The index $i = 1,2,3$ is the flavor (or generation) index. There is a single scalar multiplet:
$$\phi(1,2)_{+1/2}. \tag{1.3}$$

3. The ϕ scalar assumes a VEV,
$$\langle\phi\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \tag{1.4}$$
so that the gauge group is spontaneously broken:
$$G_{\rm SM} \to SU(3)_{\rm C} \times U(1)_{\rm EM}. \tag{1.5}$$

We now study the SM Lagrangian, that is the most general renormalizable Lagrangian that is consistent with the gauge symmetry $G_{\rm SM}$ of eq. (1.1). Let us first recall some features of gauge symmetries. Consider a Lagrangian that depends on a field Φ and its derivative $\partial_\mu\Phi$: $\mathcal{L}(\Phi, \partial_\mu\Phi)$. Under a gauge transformation,
$$\begin{aligned} \Phi(x) &\to e^{i\epsilon_a(x)T_a}\Phi(x), \\ \delta\Phi(x) &= i\epsilon_a(x)T_a\Phi(x), \end{aligned} \tag{1.6}$$

$$\begin{aligned}\partial_\mu\Phi(x) &\to e^{i\epsilon_a(x)T_a}\partial_\mu\Phi(x)+i(\partial_\mu\epsilon_a(x))T_a e^{i\epsilon_a(x)T_a}\Phi(x),\\ \delta(\partial_\mu\Phi(x)) &= i\epsilon_a(x)T_a\partial_\mu\Phi(x)+i(\partial_\mu\epsilon_a(x))T_a\Phi(x).\end{aligned}\tag{1.7}$$

Consequently, the kinetic terms in the Lagrangian, $(\partial^\mu\phi(x))^\dagger(\partial_\mu\phi(x))$ for a scalar field and $\bar\psi(x)\gamma^\mu\partial_\mu\psi(x)$ for a fermion field, are not invariant under the gauge symmetry. To have a gauge symmetry, we need to replace the derivative ∂^μ with the covariant derivative D^μ,

$$D^\mu=\partial^\mu+igT_aA_a^\mu,\tag{1.8}$$

where A_a^μ is a vector boson field in the adjoint representation of the gauge group, that is

$$\delta A_a^\mu=-f_{abc}\epsilon_b A_c^\mu-\frac{1}{g}\partial^\mu\epsilon_a.\tag{1.9}$$

Given the SM gauge group, the covariant derivative D^μ is given by

$$D^\mu=\partial^\mu+ig_sG_a^\mu L_a+igW_b^\mu T_b+ig'B^\mu Y.\tag{1.10}$$

Here G_a^μ are the eight gluon fields, W_b^μ the three weak interaction bosons and B^μ the single hypercharge boson. The L_a's are $SU(3)_{\rm C}$ generators (the 3×3 Gell-Mann matrices $\frac12\lambda_a$ for triplets, 0 for singlets), the T_b's are $SU(2)_{\rm L}$ generators (the 2×2 Pauli matrices $\frac12\tau_b$ for doublets, 0 for singlets), and Y are the $U(1)_{\rm Y}$ charges. For example, for the left-handed quarks Q_L^I, we have

$$\begin{aligned}\mathcal{L}_{\rm kin}(Q_L) &= i\overline{Q_{Li}^I}\gamma^\mu D_\mu Q_{Li}^I,\\ D^\mu Q_{Li}^I &= (\partial^\mu+\frac{i}{2}g_sG_a^\mu\lambda_a+\frac{i}{2}gW_b^\mu\tau_b+\frac{i}{6}g'B^\mu)Q_{Li}^I,\end{aligned}\tag{1.11}$$

while for the scalar doublet

$$\begin{aligned}\mathcal{L}_{\rm kin}(\phi) &= (D^\mu\phi(x))^\dagger(D_\mu\phi(x)),\\ D^\mu\phi &= (\partial^\mu+\frac{i}{2}gW_b^\mu\tau_b+\frac{i}{2}g'B^\mu)\phi.\end{aligned}\tag{1.12}$$

By (1.11) (and the analogous equations for all other fermion representations) and (1.12) we have specified the gauge interactions of the fermions and scalars. The gauge boson self interactions come from

$$\begin{aligned}\mathcal{L}_{\rm kin}(A^\mu) &= -\frac14{\rm tr}\ F^{\mu\nu_a}F_{a\mu\nu},\\ F_a^{\mu\nu} &= \partial^\mu A_a^\nu-\partial^\nu A_a^\mu-gf_{abc}A_b^\mu A_c^\nu.\end{aligned}\tag{1.13}$$

For non-Abelian symmetries, (1.13) induces trilinear and quartic self-couplings, while for Abelian symmetries (where $f_{abc}=0$) there are no vector boson self interactions.

All other terms in the Lagrangian depend on Φ (and not on $\partial_\mu\Phi$) and therefore it is enough to check that they are symmetric under a global transformation. First, there are terms that involve two fermions and a scalar, that is *Yukawa interactions*:

$$-\mathcal{L}_Y=Y_{ij}^d\overline{Q_{Li}^I}\phi d_{Rj}^I+Y_{ij}^u\overline{Q_{Li}^I}\tilde\phi u_{Rj}^I+Y_{ij}^\ell\overline{L_{Li}^I}\phi\ell_{Rj}^I.\tag{1.14}$$

Second, there are scalar self-interactions, that is the *Higgs potential*:

$$V = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2. \tag{1.15}$$

The full SM Lagrangian is then

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_Y - V. \tag{1.16}$$

To investigate the consequences of spontaneous symmetry breaking, we decompose the scalar doublet into its components:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \tag{1.17}$$

where the indices + or 0 denote electromagnetic charges, defined by $Q_{\rm EM} = T_3 + Y$. Then, using $\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}$, we find that $\mathcal{L}_{\rm kin}(\phi)$ of eq. (1.12) gives mass terms to the electroweak gauge bosons W_b^μ and B^μ:

$$\begin{aligned} &\frac{1}{8}\left| \begin{pmatrix} gW_3^\mu + g'B^\mu & g(W_1^\mu - iW_2^\mu) \\ g(W_1^\mu + iW_2^\mu) & -gW_3^\mu + g'B^\mu \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2 \\ &= \frac{1}{4} g^2 v^2 W_-^\mu W_{+\mu} + \frac{1}{8}(g^2 + g'^2) v^2 Z^\mu Z_\mu, \end{aligned} \tag{1.18}$$

where we defined

$$\begin{aligned} W_\pm^\mu &= \frac{1}{\sqrt{2}}(W_1^\mu \mp iW_2^\mu), \\ Z^\mu &= \frac{1}{\sqrt{g^2 + g'^2}}(gW_3^\mu - g'B^\mu), \\ A^\mu &= \frac{1}{\sqrt{g^2 + g'^2}}(g'W_3^\mu + gB^\mu). \end{aligned} \tag{1.19}$$

Defining

$$\tan\theta_W \equiv \frac{g'}{g}, \tag{1.20}$$

we can write the two neutral gauge bosons as

$$\begin{aligned} Z^\mu &= \cos\theta_W W_3^\mu - \sin\theta_W B^\mu, \\ A^\mu &= \sin\theta_W W_3^\mu + \cos\theta_W B^\mu. \end{aligned} \tag{1.21}$$

We identify the vector boson masses:

$$m_W^2 = \frac{1}{4} g^2 v^2, \quad m_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2, \quad m_A^2 = 0. \tag{1.22}$$

(The masslessness of the photon is not a prediction of the SM. It is guaranteed by the exact $U(1)_{\rm EM}$ gauge symmetry. It is only a consistency check of our calculations.) We also identify the ρ relation:

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1. \tag{1.23}$$

This is a prediction of all models where the $SU(2)_{\rm L} \times U(1)_{\rm Y} \to U(1)_{\rm EM}$ breaking is induced by scalar doublets only.

It will be useful to investigate the SM in the limit of vanishing Yukawa couplings, $Y^d = Y^u = Y^\ell = 0$. First, we note that there are five parameters not related to the Yukawa interactions. The gauge interactions are described by three parameters: the gauge couplings g_s, g and g'. The Higgs potential depends on two parameters, μ^2 and λ. Second, we note that in this case there is a large global symmetry:

$$G_{\rm global}(Y^f = 0) = U(3)_Q \times U(3)_{\bar d} \times U(3)_{\bar u} \times U(3)_L \times U(3)_{\bar\ell}. \tag{1.24}$$

II. PROBLEMS OF THE STANDARD MODEL

In spite of the enormous experimental success of the Standard Model (SM), it is commonly believed that it is not the full picture of Nature and that there exists New Physics (NP) beyond the SM at an energy scale higher than the electroweak breaking (EW) scale ($\Lambda_{\rm EW} \sim 10^2\ GeV$).

It is indeed clear that the SM cannot describe physics above a scale $m_{\rm Pl} \sim 10^{19}\ GeV$. At this scale, gravitational effects become as important as the known gauge interactions and cannot be neglected. But there are good reasons to believe that there is additional NP between $\Lambda_{\rm EW}$ and $m_{\rm Pl}$, Let us first list the relevant problems related to experiments and observations.

(i) There are two, related, pieces of experimental evidence for such NP. Both suggest that the neutrinos are massive, in contrast to the Standard Model prediction that they are massless. First, measurements of the flux of atmospheric neutrions find that the ratio of ν_μ-to-ν_e fluxes is different from expectations and that the ν_μ flux has an up-down asymmetry. Both facts can be beautifully explained by neutrino masses and mixing which lead to $\nu{+}\mu{-}\nu_\tau$ oscillations. Second, measurements of the solar neutrino flux find that the flux of electron-neutrinos is significantly smaller than the total flux of active ($\nu_a = \nu_e, \nu_\mu, \nu_\tau$) neutrinos. This puzzle (the Sun produces only ν_e's) can be beautifully explained by $\nu_e - \nu_a$ mixing.

(ii) There exists also an 'observational' evidence for NP. The standard Big Bang Nucleosynthesis (BBN) scenario is consistent with the observed abundance of light elements only for a certain range of baryon asymmetry. To generate baryon asymmetry, CP violation is required. The SM CP violation generates baryon asymmetry that is smaller by at least 12 orders of magnitude than the 'observed' asymmetry. This implies that there are new sources of CP violation, beyond the SM.

(iii) Another observation that cannot be explained within the Standard Model is the requirement for dark matter.

(iv) The three gauge couplings of the strong, weak and electromagnetic interactions seem to converge to a unified value at a high energy scale. The Standard Model cannot explain this fact, which is just accidental within this model.

There are other deficiencies in the SM. The most serious ones are related to problems of 'Naturalness': there are small parameters in the SM and it requires miraculous fine-tuning to explain them.

(i) The mass-squared of the Higgs gets quadratically divergent radiative corrections. This means that, if there is no New Physics below $m_{\rm Pl}$, the bare mass-squared term and the loop contributions have to cancel each other to an accuracy of about thirty four orders of magnitude. Supersymmetry (SUSY) can solve this fine-tuning problem in that is stabilizes the ratio $\Lambda_{\rm EW}/m_{\rm Pl}$. Dynamical SUSY breaking (DSB) can even explain this ratio.

(ii) The CP violating $\theta_{\rm QCD}$ parameter contributes to the electric dipole moment of the neutron. For $\theta_{\rm QCD} = \mathcal{O}(1)$ this contribution exceeds the experimental upper bound by about nine order of magnitude. This fine-tuning problem can be solved by a Peccei-Quinn symmetry, by making CP a spontaneously broken symmetry or if $m_u = 0$.

(iii) The Yukawa couplings are small (except for the top Yukawa) and hierarchical. For example, the electron Yukawa is of $\mathcal{O}(10^{-5})$. Horizontal symmetries can make the smallness and hierarchy in the flavor parameters natural.

Finally, there are considerable theoretical efforts into finding a theory that is more 'aesthetic' and capable of answering more questions than the SM. For example, string theory has, in principle, one free parameter (compared to the eighteen of the SM). It can explain, in principle, why our Universe is four-dimensional, why there are three fermion generations, etc.

Several aspects of the SM have not been tested well yet. In particular, we have little direct experimental information on the mechanism of spontaneous symmetry breaking. The hope is that the LHC experiment will find the Higgs boson and will find or seriously constrain Supersymmetry. Flavor physics and CP violation are only beginning to be tested in the B factories (BaBar and Belle). Finally, there are several on-going and forthcoming neutrino experiments that should further explore neutrino masses and mixing.

III. THE SCALE OF NEW PHYSICS

The hierarchy problem arises from the fact that there are quadratically divergent loop contributions to the Higgs mass which drive the Higgs mass to unacceptably large values unless the tree level mass parameter is finely tuned to cancel the large quantum corrections.

The most significant of these divergences come from three sources. They are - in order of decreasing magnitude - one loop diagrams involving the top quark, the electroweak gauge bosons, and the Higgs itself.

For the sake of concreteness (and, also, because this is the scale that will be probed by the LHC), let us assume that the SM is valid up to a cut-off scale of 10 TeV. Then, the contributions from the three diagrams are

$$-\frac{3}{8\pi^2}Y_t^2\Lambda^2 \sim -(2\ TeV)^2 \tag{3.1}$$

from the top loop,

$$\frac{1}{16\pi^2}g^2\Lambda^2 \sim (700\ GeV)^2 \tag{3.2}$$

from the gauge loop, and

$$\frac{1}{16\pi^2}\lambda^2\Lambda^2 \sim (500\ GeV)^2 \tag{3.3}$$

from the Higgs loop. Thus the total Higgs mass is approximately

$$m_h^2 = m_{\rm tree}^2 - [100 - 10 - 5](200\ GeV)^2. \tag{3.4}$$

In order for this to add up to a Higgs mass of order a few hundred GeV as required in the SM, fine tuning of order one part in 100 is required. This is the hierarchy problem.

Is the SM already fine tuned given that we have experimentally probed to near 1 TeV and found no NP? Setting $\Lambda = 1$ TeV in the above formulas we find that the most dangerous contribution from the top loop is only about $(200\ GeV)^2$. Thus no fine tuning is required, the SM with no NP up to 1 TeV is perfectly natural, and we should not be surprised that we have not yet seen deviations from it at colliders.

We can now turn the argument around and use the hierarchy problem to predict what forms of new physics exist at what scale in order to solve rge hierarchy problem. Consider for example the contribution to the Higgs mass from the top loop. Limiting the contribution to be no larger than about 10 times the Higgs mass (limiting fine tuning to less than 1 part in 10) we find a maximum cut-off for $\Lambda = 2$ TeV. In other words, we predict the existence of new particles with masses less than 2 TeV which cancel the quadratically divergent contribution to the Higgs mass from the top loop. In order for this cancellation to occur naturally, the new particles must be related to the top quark by a symmetry. In practice, this means that the new particles have to carry similar quantum numbers to top quarks. In supersymmetry, these new particles are of course the top squarks.

The contributions from gauge loops also need to be canceled by new particles which are related to the SM $SU(2) \times U(1)$ gauge bosons by a symmetry. The masses of these states are predicted to be at or below 5 TeV for the cancellation to be natural. Similarly, the Higgs loop requires new states related to the Higgs at 10 TeV. We see that the hierarchy problem can be used to obtain specific predictions.