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Flavor Physics and CP Violation

Yosef Nir

Department of Particle Physics
Weizmann Institute of Science, Rehovot 76100, Israel
ftnir@clever.weizmann.ac.il

This is a written version of a series of three lectures aimed at graduate students in the field of experimental high energy physics. The emphasis is on physics that is relevant to B-factories. The main topics covered are:

(i) The flavor sector of the Standard Model;
(ii) Determination of the mixing parameters;
(iii) CP violation in meson decays (a model independent description);
(iv) CP violation in the Standard Model;
(v) CP violation as a probe of new physics.

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1. Flavor Physics

1.1. What is Flavor Physics and Why is it Interesting?

The Standard Model fermions appear in three generations. *Flavor physics* describes interactions that distinguish between the fermion generations.

The fermions experience two types of interactions: gauge interactions, where two fermions couple to a gauge boson, and Yukawa interactions, where two fermions couple to a scalar. Within the Standard Model [1-3], there are twelve gauge bosons, related to the gauge symmetry

$$G_{\rm SM} = SU(3)_{\rm C} \times SU(2)_{\rm L} \times U(1)_{\rm Y}, \tag{1.1}$$

and a single Higgs scalar, related to the spontaneous symmetry breaking

$$G_{\rm SM} \to SU(3)_{\rm C} \times U(1)_{\rm EM}. \tag{1.2}$$

In the *interaction basis*, gauge interactions are diagonal (and universal, namely described by a single gauge coupling for each factor in $G_{\rm SM}$: g_s, g and g'). By definition, the interaction eigenstates have no gauge couplings between fermions of different generations. The Yuakawa interactions are, however, quite complicated in the interaction basis. In particular, there are Yukawa couplings that involve fermions of different generations and, consequently, the interaction eigenstates do not have well-defined masses. *Flavor Physics* refers to the part of the Standard Model that depends on the Yukawa couplings.

In the *mass basis*, Yukawa interactions are diagonal (though not universal). The mass eigenstates have, by definition, well-defined masses. The gauge interactions related to spontaneously broken symmetries can, however, be quite complicated in the mass basis. In particular, the $SU(2)_{\rm L}$ gauge couplings are not diagonal, that is they *mix* quarks of different generations. *Flavor Physics* here refers to fermion masses and mixings.

Why is flavor physics interesting?

(i) Flavor physics has not been well tested yet. For the gauge interactions, experiments (particularly LEP and SLD) have provided us with tests at or even below an accuracy level of one percent, where radiative corrections become essential. In contrast, several flavor parameters are only known to an accuracy level of $\mathcal{O}(30\%)$. Many rare decay processes that are sensitive to the flavor parameters have not been measured yet. In the near future, various experiments (particularly CLEO, BaBar and Belle) will substantially improve the determination of the flavor parameters and will measure various rare B decays, thus providing much more stringent tests of this sector of the Standard Model.

(ii) Most of the Standard Model flavor parameters are small and hierarchical. The Standard Model does not provide any explanation of these features. This is *the flavor puzzle*. It may be a hint for physics beyond the Standard Model, where the smallness and hierarchy of the flavor parameters find a natural explanation. For example, horizontal symmetries (that is, symmetries under which different generations transform differently) that are broken by a small parameter give selection rules for the Yukawa couplings.

(iii) Flavor changing neutral current processes (FCNC) depend on the flavor parameters. For vanishing Yukawa couplings, FCNC would be absent to all orders in the gauge couplings. Consequently, within the Standard Model FCNC are suppressed by small mixing angles and, in some cases, small quark masses. Furthermore, within the Standard Model FCNC vanish at tree level. Consequently, they are further suppressed by powers of the weak coupling. Many extensions of the Standard Model allow significant new contributions to these processes that modify the Standard Model predictions. Therefore, the flavor sector is a very sensitive probe of New Physics.

(iv) CP violation is closely related to flavor physics. It is one of the least tested aspects of the Standard Model: Even though the Kobayashi-Maskawa phase [4] can account for the CP violation that has been measured in K decays [5], the Standard Model picture of CP violation could still be completely wrong. Almost any extension of the Standard Model provides new sources of CP violation. The observed baryon asymmetry of the universe *requires* new sources of CP violation [6]. (The motivation to study CP violation is described in more detail in section 3.)

1.2. What are the Flavor Parameters?

The Standard Model fermions appear in three generations. Each generation is made of five different representations of the Standard Model gauge group $G_{\rm SM}$ of eq. (1.1):

$$Q^I_{Li}(3,2)_{+1/6},\quad u^I_{Ri}(3,1)_{+2/3},\quad d^I_{Ri}(3,1)_{-1/3},\quad L^I_{Li}(1,2)_{-1/2},\quad \ell^I_{Ri}(1,1)_{-1}. \tag{1.3}$$

Our notations mean that, for example, the left-handed quarks, Q_L^I, are in a triplet (3) of the $SU(3)_{\rm C}$ group, a doublet (2) of $SU(2)_{\rm L}$ and carry hypercharge $Y = Q_{\rm EM} - T_3 = +1/6$. The index I denotes *interaction eigenstates.* The index $i = 1, 2, 3$ is the *flavor* (or generation) index. (The above representations describe quarks and leptons and include, therefore, left-handed and right-handed fields. An alternative way to write down the various representations, which is particularly useful for the supersymmetric extension of the Standard Model, is to describe left-handed fields only. Now the fields include also antiquarks and antileptons:

$$Q_i^I(3,2)_{+1/6}, \quad \bar{u}_i^I(\bar{3},1)_{-2/3}, \quad \bar{d}_i^I(\bar{3},1)_{+1/3}, \quad L_i^I(1,2)_{-1/2}, \quad \bar{\ell}_i^I(1,1)_{+1}.) \tag{1.4}$$

The Standard Model gauge interactions do not distinguish between the different generations. Another way to state this is to say that the gauge interactions are flavor-blind. The strength of the gauge interactions depends on the gauge quantum numbers given in (1.3) and not on the flavor index i. Most important for our purposes, the interaction of the $SU(2)_L$ gauge bosons (W_μ^a, $a = 1, 2, 3$) with quarks is given by

$$-\mathcal{L}_W = \frac{g}{2} \overline{Q_{Li}^I} \gamma^\mu \tau^a Q_{Li}^I W_\mu^a. \tag{1.5}$$

The 4×4 matrix γ^μ operates in Lorentz space (it describes the combination of two spin-1/2 quark fields and one spin-1 gauge boson field into a Lorentz scalar) and the 2×2 matrix τ^a operates in the $SU(2)_{\rm L}$ space (it describes the combination of the two quark doublets and the W^a-triplet into an $SU(2)_{\rm L}$ singlet). The coupling $\overline{Q_{Li}^I} Q_{Li}^I$ can be equivalently written as $\overline{Q_{Li}^I} \mathbf{1}_{ij} Q_{Lj}^I$ where the 3×3 unit matrix $\mathbf{1}$ operates in flavor space and makes the universality of the gauge interactions manifest.

The Yukawa interactions have a complicated form in this basis:

$$-\mathcal{L}_Y = Y_{ij}^d \overline{Q_{Li}^I} \phi d_{Rj}^I + Y_{ij}^u \overline{Q_{Li}^I} \tilde{\phi} u_{Rj}^I + Y_{ij}^\ell \overline{L_{Li}^I} \phi \ell_{Rj}^I, \tag{1.6}$$

where $\phi(1,2)_{+1/2}$ is the Standard Model Higgs doublet, and $\tilde{\phi} = i\sigma_2 \phi^*$. The Yukawa matrices Y^d, Y^u and Y^ℓ are general (and, in particular, complex) 3×3 matrices. Note that, in the absence of right-handed neutrinos, $N_i(1,1)_0$, one cannot write (renormalizable) Yukawa interactions for the neutrinos.

To transform to the mass basis, one has to take into account spontaneous symmetry breaking (1.2). Within the Standard Model this breaking is the result of a vacuum expectation value assumed by the neutral component of the Higgs doublet, $\langle\phi^0\rangle = \frac{v}{\sqrt{2}}$ with the electroweak breaking scale of order $v \approx 246\ GeV$. Upon the replacement $\mathcal{R}e(\phi^0) \to (v + H^0)/\sqrt{2}$, the Yukawa interactions (1.6) give rise to mass terms:

$$-\mathcal{L}_M = (M_d)_{ij}\overline{d^I_{Li}}d^I_{Rj} + (M_u)_{ij}\overline{u^I_{Li}}u^I_{Rj} + (M_\ell)_{ij}\overline{\ell^I_{Li}}\ell^I_{Rj}, \tag{1.7}$$

where

$$M_f = \frac{v}{\sqrt{2}}Y^f, \tag{1.8}$$

and we decomposed the $SU(2)_{\rm L}$ doublets into their components:

$$Q^I_{Li} = \begin{pmatrix} u^I_{Li} \\ d^I_{Li} \end{pmatrix}, \quad L^I_{Li} = \begin{pmatrix} \nu^I_{Li} \\ \ell^I_{Li} \end{pmatrix}. \tag{1.9}$$

Since neutrinos have no Yukawa interactions, they are massless.

The mass basis corresponds, by definition, to diagonal mass matrices. We can always find unitary matrices V_{fL} and V_{fR} such that

$$V_{fL}M_fV^\dagger_{fR} = M^{\rm diag}_f, \tag{1.10}$$

with $M^{\rm diag}_f$ diagonal and real. The mass eigenstates are then identified as

$$\begin{aligned} d_{Li} &= (V_{dL})_{ij}d^I_{Lj}, \quad d_{Ri} = (V_{dR})_{ij}d^I_{Rj}, \\ u_{Li} &= (V_{uL})_{ij}u^I_{Lj}, \quad u_{Ri} = (V_{uR})_{ij}u^I_{Rj}, \\ \ell_{Li} &= (V_{\ell L})_{ij}\ell^I_{Lj}, \quad \ell_{Ri} = (V_{\ell R})_{ij}\ell^I_{Rj}, \\ \nu_{Li} &= (V_{\nu L})_{ij}\nu^I_{Lj}. \end{aligned} \tag{1.11}$$

Note that, since the neutrinos are massless, $V_{\nu L}$ is arbitrary.

The charged current interactions (that is the interactions of the charged $SU(2)_{\rm L}$ gauge bosons $W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)$), which in the interaction basis are described by (1.5), have a complicated form in the mass basis:

$$-\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}}\overline{u_{Li}}\gamma^\mu(V_{uL}V^\dagger_{dL})_{ij}d_{Lj}W^+_\mu + \text{h.c.}. \tag{1.12}$$

The 3×3 unitary matrix,

$$V_{\rm CKM} = V_{uL} V_{dL}^{\dagger}, \tag{1.13}$$

is the CKM *mixing matrix* for quarks [7,4]. It generally depends on nine parameters: three real angles and six phases.

The form of the matrix is not unique. Usually, the following two conventions are employed:

(i) There is freedom in defining $V_{\rm CKM}$ in that we can permute between the various generations. This freedom is fixed by ordering the up quarks and the down quarks by their masses, i.e. $m_{u_1} < m_{u_2} < m_{u_3}$ and $m_{d_1} < m_{d_2} < m_{d_3}$. (Usually, we call $(u_1, u_2, u_3) \to (u, c, t)$ and $(d_1, d_2, d_3) \to (d, s, b)$.) It is an interesting fact that with this convention $V_{\rm CKM}$ is close to a unit matrix. (See, for example, ref. [8] for a discussion of this point in the framework of horizontal symmetries.)

(ii) There is further freedom in the phase structure of $V_{\rm CKM}$. Let us define P_f ($f = u, d, \ell$) to be diagonal unitary (phase) matrices. Then, if instead of using V_{fL} and V_{fR} for the rotation (1.11) to the mass basis we use $\tilde{V}_{fL}$ and $\tilde{V}_{fR}$, defined by $\tilde{V}_{fL} = P_f V_{fL}$ and $\tilde{V}_{fR} = P_f V_{fR}$, we still maintain a legitimate mass basis since $M_f^{\rm diag}$ remains unchanged by such transformations. However, $V_{\rm CKM}$ does change:

$$V_{\rm CKM} \to P_u V_{\rm CKM} P_d^*. \tag{1.14}$$

This freedom is fixed by demanding that $V_{\rm CKM}$ will have the minimal number of phases. In the three generation case $V_{\rm CKM}$ has a single phase. (There are five phase differences between the elements of P_u and P_d and, therefore, five of the six phases in the CKM matrix can be removed.) This is the Kobayashi-Maskawa phase $\delta_{\rm KM}$ which is the single source of *CP violation* in the Standard Model [4].

As a result of the fact that $V_{\rm CKM}$ is not diagonal, the $W^{\pm}$ gauge bosons can couple to quark (mass eigenstates) of different generations. Within the Standard Model, this is the only source of *flavor changing* interactions. In principle, there could be additional sources of flavor mixing in the lepton sector and in Z^0 interactions. We now explain why, within the Standard Model, this does not happen.

Mixing in the lepton sector: An analysis similar to the above applies also to the left-handed leptons. The mixing matrix is $(V_{\nu L} V_{\ell L}^{\dagger})$. However, we can use the arbitrariness of $V_{\nu L}$ (related to the masslessness of neutrinos) to choose $V_{\nu L} = V_{\ell L}$, and the mixing matrix becomes a unit matrix. We conclude that the masslessness of neutrinos (if true) implies that there is no mixing in the lepton sector. If neutrinos have masses then the leptonic charged current interactions will exhibit mixing and CP violation.

Mixing in neutral current interactions: Defining $\tan\theta_W \equiv g'/g$, the Standard Model gives

$$Z^{\mu} = \cos\theta_W W_3^{\mu} - \sin\theta_W B^{\mu}. \tag{1.15}$$

(B is the gauge boson related to $U(1)_{\mathrm{Y}}$.) Therefore, to study the interactions of the Z boson, we need to know the W_3-interactions (given in (1.5)) and the B interactions:

$$-\mathcal{L}_B = -g' \left[\frac{1}{6} \overline{Q_{Li}^I} \gamma^{\mu} \mathbf{1}_{ij} Q_{Lj}^I + \frac{2}{3} \overline{u_{Ri}^I} \gamma^{\mu} \mathbf{1}_{ij} u_{Rj}^I - \frac{1}{3} \overline{d_{Ri}^I} \gamma^{\mu} \mathbf{1}_{ij} d_{Rj}^I \right] B_{\mu}. \tag{1.16}$$

Let us examine, for example, the Z-interactions with d_L in the mass basis:

$$\begin{aligned} -\mathcal{L}_Z =& \frac{g}{\cos\theta_W} \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W \right) \overline{d_{Li}} \gamma^{\mu} (V_{dL}^{\dagger} V_{dL})_{ij} d_{Lj} Z_{\mu} \\ =& \frac{g}{\cos\theta_W} \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W \right) \overline{d_{Li}} \gamma^{\mu} d_{Li} Z_{\mu}. \end{aligned} \tag{1.17}$$

We learn that the neutral current interactions remain universal in the mass basis and there are no additional flavor parameters in their description. This situation goes beyond the Standard Model to all models where all left-handed quarks are in $SU(2)_{\mathrm{L}}$ doublets and all right-handed ones in singlets. The Z-boson does have flavor changing couplings in models where this is not the case.

How many flavor parameters are there in the Standard Model? In the interaction basis, the flavor parameters come from the three Yukawa matrices. Since each of these is a 3×3 complex matrix, there are 27 real and 27 imaginary parameters in these matrices. Not all of them are, however, physical. If we switch off the Yukawa matrices, there is a global symmetry added to the Standard Model,

$$G_{\mathrm{global}}(Y^f = 0) = U(3)_Q \times U(3)_{\bar{d}} \times U(3)_{\bar{u}} \times U(3)_L \times U(3)_{\bar{\ell}}. \tag{1.18}$$

A unitary rotation of the three generations for each of the five representations in (1.3) would leave the Standard Model Lagrangian invariant. This means that the physics described by a given set of Yukawa matrices (Y^d, Y^u, Y^ℓ), and the physics described by another set,

$$\tilde{Y}^d = V_Q^\dagger Y^d V_{\bar{d}}, \quad \tilde{Y}^u = V_Q^\dagger Y^u V_{\bar{u}}, \quad \tilde{Y}^\ell = V_L^\dagger Y^\ell V_{\bar{\ell}}, \tag{1.19}$$

where V are all unitary matrices, is the same. One can use this freedom to remove, at most, 15 real and 30 imaginary parameters (the number of parameters in five 3×3 unitary matrices). However, the fact that the Standard Model with the Yukawa matrices switched on has still a global symmetry of

$$G_{\rm global} = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau \tag{1.20}$$

means that only 26 imaginary parameters can be removed. We conclude that there are 13 flavor parameters: 12 real ones and a single phase.

Examining the mass basis one can easily identify the flavor parameters. In the quark sector, we have six quark masses, three mixing angles (the number of real parameters in $V_{\rm CKM}$) and the single phase $\delta_{\rm KM}$ mentioned above. In the lepton sector, we have the three charged lepton masses.

2. The Mixing Parameters

While the fermion masses are determined from kinematics of various processes so that the values are model independent, the mixing parameters can only be determined from weak interaction processes and could be affected by new physics. There is an intensive experimental effort to measure the elements of the CKM matrix,

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{2.1}$$

There are three ways to determine the CKM parameters:

(i) Direct measurements: Standard Model tree level processes;

(ii) Unitarity: relations among the CKM elements following from $V_{\rm CKM}^\dagger V_{\rm CKM} = \mathbf{1}$;

(iii) Indirect measurements: Standard Model loop processes.

Direct measurements are expected to hold almost model independently. The reason is that viable extensions of the Standard Model have built-in mechanisms to suppress flavor changing processes in order that the strong constraints from FCNC are satisfied. These mechanisms make the contributions to Standard Model tree level processes highly suppressed. In most extensions of the Standard Model, the new physics takes place at a scale $\Lambda_{\rm NP}$ that is much higher than the EW breaking scale and consequently the contributions to decay amplitudes are suppressed by $\mathcal{O}(m_Z^2/\Lambda_{\rm NP}^2) \ll 1$. In the next subsection we briefly describe the determination of CKM elements by direct measurements.

Unitarity holds if the only quarks (namely, color triplets with electric charges $+2/3$ or $-1/3$) are those of the three generations of the Standard Model. In many extensions of the Standard Model, *e.g.* the minimal supersymmetric extension of the Standard Model, this is indeed the situation and unitarity is a valid way of determining CKM elements.

If there are additional quarks, which could be either sequential (fourth generation) or non-sequential (*e.g.* vector-like down quarks $D(3,1)_{-1/3} + \bar{D}(\bar{3},1)_{+1/3}$), and if these extra quarks mix with the observed quarks, then CKM unitarity is violated.

Indirect measurements are very sensitive to new physics. Take, for example, the $B-\bar{B}$ mixing amplitude. Within the Standard Model, the leading contribution comes from an EW box diagram and is therefore of $\mathcal{O}(g^4)$ and depends on small mixing angles, $|V_{tb}V_{td}|^2$. These suppression factors do not necessarily persist in extensions of the Standard Model. For example, in supersymmetric models there could be contributions of $\mathcal{O}(g_s^4)$ (gluino-mediated) and the mixing angles could be comparable to (or even larger than) the Standard Model ones. The validity of indirect measurements is then model dependent.

One can make however a generic statement about the relation between violation of CKM unitarity and the validity of indirect measurements [9]. Let us consider again the measurement of $|V_{tb}V_{td}|$ from Δm_B. In models with vector-like down quarks, there is a *tree level* (Z-mediated) contribution to this amplitude. In four generation models, the heavy mass of the t' quark gives an enhancement factor. In either case, whenever there is a non-negligible violation of the CKM unitarity, there will be a much more significant modification of the Standard Model predictions through large contributions to FCNC processes.

The most efficient way to investigate the mixing parameters is then the following:

(1) Measure as many parameters as possible by direct measurements. At present we have $|V_{ud}|$, $|V_{us}|$, $|V_{ub}|$, $|V_{cd}|$, $|V_{cs}|$, $|V_{cb}|$ and $|V_{tb}|$.

(2) Test whether the directly measured elements are consistent with unitarity. If there is consistency, determine the 'missing' parameters (or improve the determination of those measured with large errors) by using unitarity. At present we do so for $|V_{td}|$, $|V_{ts}|$, $|V_{tb}|$ and $|V_{cs}|$. If there is inconsistency, then most likely the quark sector extends beyond the three generations of the Standard Model.

(3) Test the predictions for FCNC processes. If there is consistency, one can further improve the determination of poorly known CKM parameters. This is the case at present for $|V_{tb}V_{td}|$ (from Δm_B and Δm_{B_s}) and for δ_{KM} (from ε_K). If there is inconsistency, then New Physics has been discovered.

2.1. Direct Measurements

Seven of the nine absolute values of the CKM entries are measured directly, namely by tree level processes. (All numbers below are taken from [10].) Nuclear beta decays give

$$|V_{ud}| = 0.9740 \pm 0.0010. \tag{2.2}$$

Semileptonic kaon and hyperon decays give

$$|V_{us}| = 0.2196 \pm 0.0023. \tag{2.3}$$

Neutrino and antineutrino production of charm off valence d quarks give

$$|V_{cd}| = 0.224 \pm 0.016. \tag{2.4}$$

Semileptonic D decays give

$$|V_{cs}| = 1.04 \pm 0.16. \tag{2.5}$$

Semileptonic exclusive and inclusive B decays give

$$|V_{cb}| = 0.0395 \pm 0.0017. \tag{2.6}$$

The endpoint spectrum in semileptonic B decays gives

$$|V_{ub}/V_{cb}| = 0.08 \pm 0.02. \tag{2.7}$$

The decay $t \to b\ell^+\nu_\ell$ gives

$$|V_{tb}|^2/(|V_{tb}|^2 + |V_{ts}|^2 + |V_{td}|^2) = 0.99 \pm 0.29. \tag{2.8}$$

2.2. Unitarity of the CKM Matrix

The requirement of CKM unitarity is simply stated as $V^\dagger_{\rm CKM} V_{\rm CKM} = \mathbf{1}$. This leads to various relations among the matrix elements. The orthogonality between any two columns will be very useful in our discussion:

$$V_{ud}V^*_{us} + V_{cd}V^*_{cs} + V_{td}V^*_{ts} = 0, \tag{2.9}$$

$$V_{us}V^*_{ub} + V_{cs}V^*_{cb} + V_{ts}V^*_{tb} = 0, \tag{2.10}$$

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0. \tag{2.11}$$

Another class of unitarity constraints is given by $\sum_{i=1}^3 |V_{ij}|^2 = \sum_{j=1}^3 |V_{ij}|^2 = 1$. A particularly useful relation is

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1. \tag{2.12}$$

Using unitarity constraints, one can narrow down some of the ranges determined from direct measurements (most noticeably, that of $|V_{cs}|$) and put constraints on the top mixings $|V_{ti}|$. For example, the relation (2.12) and the very small measured values of $|V_{ub}|$ and $|V_{cb}|$ imply that, to an excellent approximation,

$$|V_{tb}| = 1. \tag{2.13}$$

The relation (2.10) and the small measured value of $|V_{us}V_{ub}|$ imply that, to a good approximation,

$$|V_{ts}| \approx |V_{cb}|. \tag{2.14}$$

The relation (2.11), together with $|V_{ub}/V_{cb}| \leq 0.10$ and $|V_{cd}/V_{ud}| = 0.22$, gives

$$|V_{td}V_{tb}| \approx 0.0085 \pm 0.0045. \tag{2.15}$$

The full information on the absolute values of the CKM elements from both direct measurements and three generation unitarity is summarized by [10]:

$$|V| = \begin{pmatrix} 0.9745-0.9760 & 0.217-0.224 & 0.0018-0.0045 \\ 0.217-0.224 & 0.9737-0.9753 & 0.036-0.046 \\ 0.004-0.013 & 0.035-0.042 & 0.9991-0.9994 \end{pmatrix}. \tag{2.16}$$

The unitarity of the CKM matrix is manifest using an explicit parameterization. There are various useful ways to parameterize it, but the standard choice [10] is the following [11]:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{2.17}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. A test of the CKM picture is then whether there is a range for the *four* parameters s_{12}, s_{23}, s_{13} and δ that is consistent with the *seven* direct measurements described in the previous subsection. Indeed, the following ranges are consistent with (2.16):

$$s_{12} = 0.2196 \pm 0.0023, \quad s_{23} = 0.0395 \pm 0.0017, \quad s_{13}/s_{23} = 0.08 \pm 0.02. \tag{2.18}$$

(The phase δ is not constrained at present by direct measurements.) Another useful parametrization is in terms of the four Wolfenstein parameters (λ, A, ρ, η) with $\lambda = |V_{us}| = 0.22$ playing the role of an expansion parameter and η representing the CP violating phase [12]:

$$V = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \tag{2.19}$$

Because of the smallness of λ and the fact that for each element the expansion parameter is actually λ^2, it is sufficient to keep only the first few terms in this expansion. The ranges in (2.16) can be translated into the following ranges of the Wolfenstein parameters:

$$\lambda = 0.2196 \pm 0.0023, \quad A = 0.819 \pm 0.035, \quad (\rho^2+\eta^2)^{1/2} = 0.36 \pm 0.09. \tag{2.20}$$

The relation between the parameters of (2.17) and (2.19) is given by

$$s_{12} \equiv \lambda, \quad s_{23} \equiv A\lambda^2, \quad s_{13}e^{-i\delta} \equiv A\lambda^3(\rho-i\eta). \tag{2.21}$$

This specifies the higher order terms in (2.19).

2.3. Neutral Meson Mixing

The presently useful indirect measurements (Δm_B, Δm_{B_s} and ε_K) are all related to neutral meson mixing. Before presenting the implications of these measurements for the CKM parameters, we briefly discuss then the physics and formalism of neutral meson mixing. We refer specifically to the neutral B meson system, but most of our discussion applies equally well to the neutral K, B_s and D systems.

Our phase convention for the CP transformation law of the neutral B mesons is defined by

$$\mathrm{CP}|B^0\rangle = \omega_B|\bar{B}^0\rangle, \quad \mathrm{CP}|\bar{B}^0\rangle = \omega_B^*|B^0\rangle, \quad (|\omega_B| = 1). \tag{2.22}$$

Physical observables do not depend on the phase factor ω_B. An arbitrary linear combination of the neutral B-meson flavor eigenstates,

$$a|B^0\rangle + b|\bar{B}^0\rangle, \tag{2.23}$$

is governed by a time-dependent Schrödinger equation,

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = H\begin{pmatrix} a \\ b \end{pmatrix} \equiv \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} a \\ b \end{pmatrix}, \tag{2.24}$$

for which M and Γ are 2×2 Hermitian matrices.

The off-diagonal terms in these matrices, M_{12} and Γ_{12}, are particularly important in the discussion of mixing and CP violation. M_{12} is the dispersive part of the transition amplitude from B^0 to $\bar{B}^0$. In the Standard Model it arises only at order g^4. In the language of quark diagrams, the leading contribution is from box diagrams. At sufficiently high loop momentum, $k \gg \Lambda_{\mathrm{QCD}}$, these diagrams are a very good approximation to the Standard Model contribution to M_{12}. This, or any other contribution from heavy intermediate states from new physics, is the *short distance* contribution. For small loop momenta, $k \lesssim 1\ GeV$, we do not expect quark hadron duality to hold. The box diagram is a poor approximation to the contribution from light intermediate states, namely to *long distance* contributions. Fortunately, in the B and B_s systems, the long distance contributions are expected to be negligible. (This is not the case for K and D mesons. Consequently, it is difficult to extract useful information from the measurement of Δm_K and from the bound on Δm_D.) Γ_{12} is the absorptive part of the transition amplitude. Since the cut of a diagram always involves on-shell particles and thus long distance physics, the cut of the quark box diagram is a poor approximation to Γ_{12}. However, it does correctly give the suppression from small electroweak parameters such as the weak coupling. In other words, though the hadronic uncertainties are large and could change the result by order 50%, the cut in the box diagram is expected to give a reasonable order of magnitude estimate of Γ_{12}. (For $\Gamma_{12}(B_s)$ it has been shown that local quark-hadron duality holds exactly in the simultaneous limit of small velocity and large number of colors. We thus expect an uncertainty of $\mathcal{O}(1/N_C) \sim 30\%$ [13-14]. For $\Gamma_{12}(B_d)$ the small velocity limit is not as good an approximation but an uncertainty of order 50% still seems a reasonable estimate.) New physics is not expected to affect Γ_{12} significantly because it usually takes place at a high energy scale and is relevant to the short distance part only.

The light B_L and heavy B_H mass eigenstates are given by

$$|B_{L,H}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle. \tag{2.25}$$

The complex coefficients q and p obey the normalization condition $|q|^2 + |p|^2 = 1$. Note that $\arg(q/p^*)$ is just an overall common phase for $|B_L\rangle$ and $|B_H\rangle$ and has no physical significance. The mass difference and the width difference between the physical states are given by

$$\Delta m \equiv M_H - M_L, \quad \Delta\Gamma \equiv \Gamma_H - \Gamma_L. \tag{2.26}$$

Solving the eigenvalue equation gives

$$\begin{aligned} (\Delta m)^2 - \frac{1}{4}(\Delta\Gamma)^2 &= (4|M_{12}|^2 - |\Gamma_{12}|^2), \\ \Delta m \Delta\Gamma &= 4\mathcal{R}e(M_{12}\Gamma_{12}^*), \end{aligned} \tag{2.27}$$

$$\frac{q}{p} = -\frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - \frac{i}{2}\Delta\Gamma} = -\frac{\Delta m - \frac{i}{2}\Delta\Gamma}{2M_{12} - i\Gamma_{12}}. \tag{2.28}$$

In the B system, $|\Gamma_{12}| \ll |M_{12}|$ (see discussion below), and then, to leading order in $|\Gamma_{12}/M_{12}|$, (2.27) and (2.28) can be written as

$$\Delta m_B = 2|M_{12}|, \quad \Delta\Gamma_B = 2\mathcal{R}e(M_{12}\Gamma_{12}^*)/|M_{12}|, \tag{2.29}$$

$$\frac{q}{p} = -\frac{M_{12}^*}{|M_{12}|}. \tag{2.30}$$

2.4. Indirect Measurements

The most useful CP conserving indirect measurement is that of Δm_B [15]:

$$\Delta m_B = 0.471 \pm 0.016 \text{ ps}^{-1}. \tag{2.31}$$

The Standard Model accounts for $\Delta m_B = 2|M_{12}|$ by box diagrams with intermediate top quarks [16]:

$$\Delta m_B = \frac{G_F^2}{6\pi^2}\eta_B m_B m_W^2 (B_B f_B^2) S_0(x_t)|V_{tb}V_{td}^*|^2, \tag{2.32}$$

where G_F is the Fermi constant, η_B is a QCD correction factor calculated in NLO [17], $S_0(x_t)$ is a kinematic function calculated from the box graphs [16], and $x_t = \bar{m}_t^2/m_W^2$. We use [18] $\overline{m_t}(m_t) = 167 \pm 6\ GeV$, giving $S_0(x_t) \approx 2.36$. $B-\bar{B}$ mixing is dominated by short distance physics (an intermediate top), so that the main source of theoretical uncertainty lies in the matrix element of the four quark operator between the meson states. The value of the matrix element is parameterized by $B_B f_B^2$ and is estimated by *e.g.* lattice calculations [10], $B_B f_B^2 = (1.4 \pm 0.1)(175 \pm 25\ MeV)^2$. The constraint on the CKM parameters from (2.32) can be written as

$$|V_{tb}^* V_{td}| = 0.0086 \left[\frac{\Delta m_B}{0.471 \text{ ps}^{-1}}\right]^{1/2} \left[\frac{0.2GeV}{\sqrt{B_B} f_B}\right] \left[\frac{2.4}{S_0(x_t)}\right]^{1/2} \left[\frac{0.55}{\eta_B}\right]^{1/2}, \tag{2.33}$$

This constraint gives at present

$$|V_{tb}^* V_{td}| = 0.0084 \pm 0.0018, \tag{2.34}$$

which is consistent with, and actually significantly improves the unitarity constraint (2.15).

Another useful indirect measurement if that of Δm_{B_s}. The expression for Δm_{B_s} is very similar to (2.31), except for the CKM dependence and an $SU(3)$ breaking factor (that is an approximate global symmetry of the strong interactions that holds in the limit $m_u = m_d = m_s = 0$):

$$\frac{\Delta m_{B_d}}{\Delta m_{B_s}} = \frac{m_{B_d}}{m_{B_s}} \frac{B_{B_d} f_{B_d}^2}{B_{B_s} f_{B_s}^2}\ |V_{td}/V_{ts}|^2. \tag{2.35}$$

The uncertainty in the ratio between the matrix elements is smaller than the uncertainty in each of them separately [19]:

$$\frac{B_{B_s} f_{B_s}^2}{B_{B_d} f_{B_d}^2} = 1.30 \pm 0.18. \tag{2.36}$$

At present, there is only a lower bound [15], $\Delta m_{B_s} \geq 12.4 \text{ ps}^{-1}$, leading to

$$|V_{td}/V_{ts}| \leq 0.24 \implies |V_{td}| \leq 0.0096, \tag{2.37}$$

which further improves the upper bound of (2.34).

The imaginary part of the $K - \bar{K}$ mixing amplitude corresponds to the CP violating observable ε_K discussed in the next chapter:

$$\varepsilon_K = \frac{\exp(i\pi/4)}{\sqrt{2}} \frac{\mathcal{I}m\ M_{12}}{\Delta m_K}. \tag{2.38}$$

The off-diagonal mass matrix element M_{12} is obtained from the $\Delta S = 2$ effective Hamiltonian with contributions from both the c-quark and the t-quark in the EW loop, yielding

$$\begin{aligned} M_{12} =& \frac{G_F^2}{12\pi^2} f_K^2 B_K m_K m_W^2 \\ &\times \left[(V_{cd}^* V_{cs})^2 \eta_1 S_0(x_c) + (V_{td}^* V_{ts})^2 \eta_2 S_0(x_t) + 2(V_{cd}^* V_{cs})(V_{td}^* V_{ts}) \eta_3 S_0(x_c, x_t) \right], \end{aligned} \tag{2.39}$$

where f_K is the kaon decay constant and η_i are QCD factors calculated in NLO [17,20]. $\mathcal{I}m\ M_{12}$ is dominated by short distance physics (intermediate top quark), so that the main source of theoretical uncertainty lies in the matrix element [19], $B_K = 0.6 - 1$. The resulting constraint on the CKM parameters can be written (with the convention that $(V_{ud}^* V_{us})$ is real) as

$$\begin{aligned} \varepsilon_K =& \exp(i\pi/4) C_{\varepsilon_K} B_K \mathcal{I}m(V_{td}^* V_{ts}) \\ &\times \{ \mathcal{R}e(V_{cd}^* V_{cs}) [\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)] - \mathcal{R}e(V_{td}^* V_{ts}) \eta_2 S_0(x_t) \}, \end{aligned} \tag{2.40}$$

where all well-known quantities have been combined in the numerical constant,

$$C_{\varepsilon_K} = \frac{G_F^2}{6\sqrt{2}\pi^2} \frac{f_K^2 m_K m_W^2}{\Delta m_K} = 3.78 \times 10^4. \tag{2.41}$$

In the future, we may get useful information about the CKM parameters from the two rare kaon decays, $K^+ \to \pi^+ \nu \bar{\nu}$ [21] and $K_L \to \pi^0 \nu \bar{\nu}$ [22], which are theoretically very clean. Both modes are dominated by short distance Z-penguins and box diagrams. The branching ratio for $K^+ \to \pi^+ \nu \bar{\nu}$ can be expressed in terms of ρ and η [18]:

$$BR(K^+ \to \pi^+ \nu \bar{\nu}) = 8.33 \times 10^{-6} |V_{cb}|^4 [X(x_t)]^2 \left[\eta^2 + (\rho_0 - \rho)^2 \right], \tag{2.42}$$

where

$$\rho_0 = 1 + \frac{P_0(X)}{X(x_t)} \frac{\lambda^4}{|V_{cb}|^2}, \tag{2.43}$$

and $X(x_t)$ and $P_0(X)$ represent the electroweak loop contributions in NLO for the top quark and for the charm quark, respectively. The main theoretical uncertainty is related to the strong dependence of the charm contribution on the renormalization scale and the QCD scale, $P_0(X) = 0.40 \pm 0.06$. First evidence for $K^+ \to \pi^+ \nu\bar{\nu}$ was presented recently [23]. The large experimental error does not yet give a useful CKM constraint and is consistent with the Standard Model prediction.

The $K_L \to \pi^0 \nu\bar{\nu}$ decay is CP violating and will be discussed later in detail. The branching ratio can be expressed in terms of η [18]:

$$BR(K_L \to \pi^0 \nu\bar{\nu}) = 3.29 \times 10^{-5} |V_{cb}|^4 [X(x_t)]^2 \eta^2. \tag{2.44}$$

The present experimental bound, $BR(K_L \to \pi^0 \nu\bar{\nu}) \leq 1.6 \times 10^{-6}$ [24] lies about five orders of magnitude above the Standard Model prediction [25] and about two orders of magnitude above the bound that can be deduced using model independent isospin relations [26] from the experimental upper bound on the charged mode.

2.5. The Unitarity Triangle

Each of the three relations (2.9)-(2.11) requires the sum of three complex quantities to vanish and so can be geometrically represented in the complex plane as a triangle. These are "the unitarity triangles", though the term "unitarity triangle" is usually reserved for the relation (2.11) only. It is a surprising feature of the CKM matrix that all unitarity triangles are equal in area. For any choice of $i, j, k, l = 1, 2, 3$, one can define a quantity J according to [27]

$$\mathcal{I}m[V_{ij} V_{kl} V_{il}^* V_{kj}^*] = J \sum_{m,n=1}^{3} \epsilon_{ikm} \epsilon_{jln}. \tag{2.45}$$

Then, the area of each unitarity triangle equals $|J|/2$ while the sign of J gives the direction of the complex vectors around the triangles. As will be discussed below, CP is violated in the Standard Model only if $J \neq 0$. The area of the triangles is then related to the size of the Standard Model CP violation.

The rescaled unitarity triangle is derived from (2.11) by (a) choosing a phase convention such that $(V_{cd}V_{cb}^*)$ is real, and (b) dividing the lengths of all sides by $|V_{cd}V_{cb}^*|$. Step (a) aligns one side of the triangle with the real axis, and step (b) makes the length of this side 1. The form of the triangle is unchanged. Two vertices of the rescaled unitarity triangle are thus fixed at (0,0) and (1,0). The coordinates of the remaining vertex correspond to the Wolfenstein parameters (ρ,η) (see (2.19)).

Depicting the rescaled unitarity triangle in the (ρ,η) plane, the lengths of the two complex sides are

$$R_u \equiv \sqrt{\rho^2+\eta^2} = \frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|, \quad R_t \equiv \sqrt{(1-\rho)^2+\eta^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|. \tag{2.46}$$

The three angles of the unitarity triangle are denoted by α,β and γ [28]:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \quad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \tag{2.47}$$

They are physical quantities and, we will soon see, can be independently measured by CP asymmetries in B decays.

The only large uncertainties in the present determination of the CKM elements are in $|V_{ub}|$ and $|V_{td}|$. However, the two are related through (2.11). Thus, the unitarity triangle is a very convenient tool for presenting constraints on these poorly determined parameters. In particular, Δm_{B_d} and Δm_{B_s} constrain R_t, the semileptonic $b\to u$ rates constrain R_u, and the ε_K constraint can be written as

$$\eta\{(1-\rho)\eta_2 S_0(x_t)|V_{cb}|^2 + \eta_3 S_0(x_c,x_t) - \eta_1 S_0(x_c)\}|V_{cb}|^2 B_K = 1.24\times 10^{-6}. \tag{2.48}$$

Examining the Wolfenstein parametrization, we learn that $|J| = \mathcal{O}(\lambda^6)\times\sin\delta$. More precisely, the ranges specified above for the mixing angles give the following 90% CL range:

$$|J| = (2.7\pm 0.7)\times 10^{-5}\ \sin\delta. \tag{2.49}$$

The measurement of ε_K is consistent with this range provided that

$$\sin\delta = \mathcal{O}(1). \tag{2.50}$$

(The phase δ is defined in eq. (2.17) and equals γ of eq. (2.47).) When all available information, including the ε_K constraint, is taken into account, we find the following allowed ranges for the CKM parameters [29-31]:

$$-0.15 \leq \rho \leq +0.35, \quad +0.20 \leq \eta \leq +0.45, \tag{2.51}$$

$$0.4 \leq \sin 2\beta \leq 0.8, \quad -0.9 \leq \sin 2\alpha \leq 1.0, \quad 0.23 \leq \sin^2 \gamma \leq 1.0. \tag{2.52}$$

3. CP Violation in Meson Decays: A Model Independent Discussion

3.1. Introduction

CP violation arises naturally in the three generation Standard Model. The CP violation that has been measured in neutral K-meson decays (ε_K) is accommodated in the Standard Model in simple way [4]. Yet, CP violation is one of the least tested aspects of the Standard Model. The value of the ε_K parameter [5] as well as bounds on other CP violating parameters (most noticeably, the electric dipole moments of the neutron, d_N, and of the electron, d_e) can be accounted for in models where CP violation has features that are very different from the Standard Model ones.

It is unlikely that the Standard Model provides the complete description of CP violation in nature. First, it is quite clear that there exists New Physics beyond the Standard Model. Almost any extension of the Standard Model has additional sources of CP violating effects. In addition there is a great puzzle in cosmology that relates to CP violation, and that is the baryon asymmetry of the universe [6]. Theories that explain the observed asymmetry must include new sources of CP violation [32]: the Standard Model cannot generate a large enough matter-antimatter imbalance to produce the baryon number to entropy ratio observed in the universe today [33-35].

In the near future, significant new information on CP violation will be provided by various experiments. The main source of information will be measurements of CP violation in various B decays, particularly neutral B decays into final CP eigenstates [36-38]. Another piece of valuable information might come from a measurement of the $K_L \to \pi^0 \nu \bar{\nu}$ decay [39,40,22,26]. For the first time, the pattern of CP violation that is predicted by the Standard Model will be tested. Basic questions such as whether CP is an approximate symmetry in nature will be answered.

It could be that the scale where new CP violating sources appear is too high above the Standard Model scale (*e.g.* the GUT scale) to give any observable deviations from the Standard Model predictions. In such a case, the outcome of the experiments will be a (frustratingly) succesful test of the Standard Model and a significant improvement in our knowledge of the CKM matrix.

A much more interesting situation will arise if the new sources of CP violation appear at a scale that is not too high above the electroweak scale. Then they might be discovered in the forthcoming experiments. Once enough independent observations of CP violating effects are made, we will find that there is no single choice of CKM parameters that is consistent with all measurements. There may even be enough information in the pattern of the inconsistencies to tell us something about the nature of the new physics contributions [9,41-43].

The aim of this and the next two chapters is to explain the theoretical tools with which we will analyze new information about CP violation. In this chapter, we give a brief, model-independent discussion of CP violating observables. In the next chapter, we discuss CP violation in the Standard Model. In the last chapter, we describe CP violation beyond the Standard Model and, in particular, in Supersymmetric models. The latter enables us to elucidate the uniqueness of the Standard Model description of CP violation and how little it has been tested so far. It further demonstrates how the information from CP violation can help us probe in detail models of New Physics.

3.2. Notations and Formalism

To understand the experimental and theoretical aspects of CP violation in meson decays, we first introduce some formalism. We continue the discussion of eqs. (2.22)-(2.27). Again, we specifically discuss the neutral B meson system, but large parts of our analysis apply equally well to the other meson systems.

To discuss CP violation in mixing (see below), it is useful to write (2.30) to first order in $|\Gamma_{12}/M_{12}|$:

$$\frac{q}{p} = -\frac{M_{12}^*}{|M_{12}|}\left[1 - \frac{1}{2}\mathcal{I}m\left(\frac{\Gamma_{12}}{M_{12}}\right)\right]. \tag{3.1}$$

To discuss CP violation in decay (see below), we need to consider decay amplitudes. The CP transformation law for a final state f is

$$\mathrm{CP}|f\rangle = \omega_f|\bar{f}\rangle, \quad \mathrm{CP}|\bar{f}\rangle = \omega_f^*|f\rangle, \quad (|\omega_f|) = 1. \tag{3.2}$$

For a final CP eigenstate $f = f_{\mathrm{CP}}$, the phase factor ω_f is replaced by $\eta_{f_{\mathrm{CP}}} = \pm 1$, the CP eigenvalue of the final state. We define the decay amplitudes A_f and $\bar{A}_f$ according to

$$A_f = \langle f|\mathcal{H}_d|B^0\rangle, \quad \bar{A}_f = \langle f|\mathcal{H}_d|\bar{B}^0\rangle, \tag{3.3}$$

where $\mathcal{H}_d$ is the decay Hamiltonian.

To discuss CP violation in the interference of decays with and without mixing (see below), we introduce a complex quantity λ_f defined by

$$\lambda_f \;=\; \frac{q}{p}\,\frac{\bar{A}_f}{A_f}. \tag{3.4}$$

The effective Hamiltonian that is relevant to M_{12} is of the form

$$H_{\mathrm{eff}}^{\Delta b=2} \propto e^{+2i\phi_B}\left[\bar{d}\gamma^\mu(1-\gamma_5)b\right]^2 + e^{-2i\phi_B}\left[\bar{b}\gamma^\mu(1-\gamma_5)d\right]^2, \tag{3.5}$$

where $2\phi_B$ is a CP violating (weak) phase. (We use the Standard Model $V-A$ amplitude, but the results can be generalized to any Dirac structure.) For the B system, where $|\Gamma_{12}| \ll |M_{12}|$, this leads to

$$q/p = \omega_B e^{-2i\phi_B}. \tag{3.6}$$

(We implicitly assumed that the vacuum insertion approximation gives the correct sign for M_{12}. In general, there is a sign(B_B) factor on the right hand side of (3.6) [44].) The decay Hamiltonian is of the form

$$H_d \propto e^{+i\phi_f}\left[\bar{q}\gamma^\mu(1-\gamma_5)d\right]\left[\bar{b}\gamma_\mu(1-\gamma_5)q\right] + e^{-i\phi_f}\left[\bar{q}\gamma^\mu(1-\gamma_5)b\right]\left[\bar{d}\gamma_\mu(1-\gamma_5)q\right], \tag{3.7}$$

where ϕ_f is the appropriate weak phase. (Again, for simplicity we use a $V-A$ structure, but the results hold for any Dirac structure.) Then

$$\bar{A}_{\bar{f}}/A_f = \omega_f\omega_B^* e^{-2i\phi_f}. \tag{3.8}$$

Eqs. (3.6) and (3.8) together imply that for a final CP eigenstate,

$$\lambda_{f_{\rm CP}} = \eta_{f_{\rm CP}} e^{-2i(\phi_B+\phi_f)}. \tag{3.9}$$

3.3. The Three Types of CP Violation in Meson Decays

There are three different types of CP violation in meson decays:

(i) CP violation in mixing, which occurs when the two neutral mass eigenstate admixtures cannot be chosen to be CP-eigenstates;

(ii) CP violation in decay, which occurs in both charged and neutral decays, when the amplitude for a decay and its CP-conjugate process have different magnitudes;

(iii) CP violation in the interference of decays with and without mixing, which occurs in decays into final states that are common to B^0 and $\bar{B}^0$. (It often occurs in combination with the other two types but there are cases when, to an excellent approximation, it is the only effect.)

(i) CP violation in mixing:

$$|q/p| \neq 1. \tag{3.10}$$

This results from the mass eigenstates being different from the CP eigenstates, and requires a relative phase between M_{12} and Γ_{12}. For the neutral B system, this effect could be observed through the asymmetries in semileptonic decays:

$$a_{\rm SL} = \frac{\Gamma(\bar{B}^0_{\rm phys}(t) \to \ell^+\nu X) - \Gamma(B^0_{\rm phys}(t) \to \ell^-\nu X)}{\Gamma(\bar{B}^0_{\rm phys}(t) \to \ell^+\nu X) + \Gamma(B^0_{\rm phys}(t) \to \ell^-\nu X)}. \tag{3.11}$$

In terms of q and p,

$$a_{\rm SL} = \frac{1-|q/p|^4}{1+|q/p|^4}. \tag{3.12}$$

CP violation in mixing has been observed in the neutral K system ($\mathcal{R}e\ \varepsilon_K \neq 0$).

In the neutral B system, the effect is expected to be small, $\lesssim \mathcal{O}(10^{-2})$. The reason is that, model independently, the effect cannot be larger than $\mathcal{O}(\Delta\Gamma_B/\Delta m_B)$. The difference in width is produced by decay channels common to B^0 and $\bar{B}^0$. The branching ratios for such channels are at or below the level of 10^{-3}. Since various channels contribute with differing signs, one expects that their sum does not exceed the individual level. Hence

$\Delta\Gamma_B/\Gamma_B = \mathcal{O}(10^{-2})$ is a rather safe and model independent assumption. On the other hand, it is experimentaly known that $\Delta m_B/\Gamma_B \approx 0.7$.

To calculate the deviation of $|q/p|$ from a pure phase (see (3.1)),

$$1 - \left|\frac{q}{p}\right| = \frac{1}{2}\mathcal{I}m\frac{\Gamma_{12}}{M_{12}}, \tag{3.13}$$

one needs to calculate M_{12} and Γ_{12}. This involves large hadronic uncertainties, in particular in the hadronization models for Γ_{12}.

(ii) CP violation in decay:

$$|\bar{A}_{\bar{f}}/A_f| \neq 1. \tag{3.14}$$

This appears as a result of interference among various terms in the decay amplitude, and will not occur unless at least two terms have different weak phases and different strong phases. CP asymmetries in charged B decays,

$$a_f = \frac{\Gamma(B^+ \to f^+) - \Gamma(B^- \to f^-)}{\Gamma(B^+ \to f^+) + \Gamma(B^- \to f^-)}, \tag{3.15}$$

are purely an effect of CP violation in decay. In terms of the decay amplitudes,

$$a_{f\pm} = \frac{1 - |\bar{A}_{f^-}/A_{f^+}|^2}{1 + |\bar{A}_{f^-}/A_{f^+}|^2}. \tag{3.16}$$

There is as yet no unambiguous experimental evidence for CP violation in decays. A measurement of $\mathcal{R}e\ \varepsilon'_K \neq 0$ [45,46] would constitute such evidence. It is also possible that the first unambiguous evidence for such CP violation will come from B decays, *e.g.* for $f^+ = \pi^+ K^0$.

There are two types of phases that may appear in A_f and $\bar{A}_{\bar{f}}$. Complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the CP-conjugate amplitude. Thus their phases appear in A_f and $\bar{A}_{\bar{f}}$ with opposite signs. In the Standard Model these phases occur only in the CKM matrix which is part of the electroweak sector of the theory, hence these are often called "weak phases". The weak phase of any single term is convention dependent. However the difference between the weak phases in two different terms in A_f is convention independent because the phase rotations of the initial and final states are the same for every term. A second type of phase can appear in scattering or decay amplitudes even when the Lagrangian is real. Such phases do not violate CP, since they appear in A_f and $\bar{A}_{\bar{f}}$ with the same sign. Their origin is the possible contribution from intermediate on-shell states in the decay process, that is an absorptive part of an amplitude that has contributions from coupled channels. Usually the dominant rescattering is due to strong interactions and hence the designation "strong phases" for the phase shifts so induced. Again only the relative strong phases of different terms in a scattering amplitude have physical content, an overall phase rotation of the entire amplitude has no physical consequences.

Thus it is useful to write each contribution to A in three parts: its magnitude A_i; its weak phase term $e^{i\phi_i}$; and its strong phase term $e^{i\delta_i}$. Then, if several amplitudes contribute to $B \to f$, we have

$$\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| = \left|\frac{\sum_i A_i e^{i(\delta_i - \phi_i)}}{\sum_i A_i e^{i(\delta_i + \phi_i)}}\right|. \tag{3.17}$$

The magnitude and strong phase of any amplitude involve long distance strong interaction physics, and our ability to calculate these from first principles is limited. Thus quantities that depend only on the weak phases are much cleaner than those that require knowledge of the relative magnitudes or strong phases of various amplitude contributions, such as CP violation in decay. There is however a large literature and considerable theoretical effort that goes into the calculation of amplitudes and strong phases . In many cases we can only relate experiment to Standard Model parameters through such calculations. The techniques that are used are expected to be more accurate for B decays than for K decays, because of the larger B mass, but theoretical uncertainty remains significant. The calculations generally contain two parts. First the operator product expansion and QCD perturbation theory are used to write any underlying quark process as a sum of local quark operators with well-determined coefficients. Then the matrix elements of the operators between the initial and final hadron states must be calculated. This is where theory is weakest and the results are most model dependent. Ideally lattice calculations should be able to provide accurate determinations for the matrix elements, and in certain cases this is already true, but much remains to be done.

(iii) CP violation in the interference between decays with and without mixing:

$$|\lambda_{f_{\rm CP}}| = 1, \quad \mathcal{I}m\ \lambda_{f_{\rm CP}} \neq 0. \tag{3.18}$$

Any $\lambda_{f_{\rm CP}} \neq \pm 1$ is a manifestation of CP violation. The special case (3.18) isolates the effects of interest since both CP violation in decay (3.14) and in mixing (3.10) lead to $|\lambda_{f_{\rm CP}}| \neq 1$. For the neutral B system, this effect can be observed by comparing decays into final CP eigenstates of a time-evolving neutral B state that begins at time zero as B^0 to those of the state that begins as $\bar{B}^0$:

$$a_{f_{\rm CP}} = \frac{\Gamma(\bar{B}^0_{\rm phys}(t) \to f_{\rm CP}) - \Gamma(B^0_{\rm phys}(t) \to f_{\rm CP})}{\Gamma(\bar{B}^0_{\rm phys}(t) \to f_{\rm CP}) + \Gamma(B^0_{\rm phys}(t) \to f_{\rm CP})}. \tag{3.19}$$

This time dependent asymmetry is given (for $|\lambda_{f_{\rm CP}}| = 1$) by

$$a_{f_{\rm CP}} = -\mathcal{I}m\lambda_{f_{\rm CP}} \sin(\Delta m_B t). \tag{3.20}$$

CP violation in the interference of decays with and without mixing has been observed for the neutral K system ($\mathcal{I}m\ \varepsilon_K \neq 0$). It is expected to be an effect of $\mathcal{O}(1)$ in various B decays. For such cases, the contribution from CP violation in mixing is clearly negligible. For decays that are dominated by a single CP violating phase (for example, $B \to \psi K_S$ and $K_L \to \pi^0 \nu\bar{\nu}$), so that the contribution from CP violation in decay is also negligible, $a_{f_{\rm CP}}$ is cleanly interpreted in terms of purely electroweak parameters. Explicitly, $\mathcal{I}m\lambda_{f_{\rm CP}}$ gives the difference between the phase of the $B-\bar{B}$ mixing amplitude ($2\phi_B$) and twice the phase of the relevant decay amplitude ($2\phi_f$) (see eq. (3.9)):

$$\mathcal{I}m\lambda_{f_{\rm CP}} = -\eta_{f_{\rm CP}} \sin[2(\phi_B + \phi_f)]. \tag{3.21}$$

3.4. The ε_K Parameter

Historically, a different language from the one used by us has been employed to describe CP violation in $K \to \pi\pi$ and $K \to \pi\ell\nu$ decays. In this section we 'translate' the language of ε_K and ε'_K to our notations. Doing so will make it easy to understand which type of CP violation is related to each quantity.

The two CP violating quantities measured in neutral K decays are

$$\eta_{00} = \frac{\langle \pi^0\pi^0|\mathcal{H}|K_L\rangle}{\langle \pi^0\pi^0|\mathcal{H}|K_S\rangle}, \quad \eta_{+-} = \frac{\langle \pi^+\pi^-|\mathcal{H}|K_L\rangle}{\langle \pi^+\pi^-|\mathcal{H}|K_S\rangle}. \tag{3.22}$$

Define

$$\begin{aligned} A_{00} &= \langle \pi^0\pi^0|\mathcal{H}|K^0\rangle, \quad \bar{A}_{00} = \langle \pi^0\pi^0|\mathcal{H}|\bar{K}^0\rangle, \\ A_{+-} &= \langle \pi^+\pi^-|\mathcal{H}|K^0\rangle, \quad \bar{A}_{+-} = \langle \pi^+\pi^-|\mathcal{H}|\bar{K}^0\rangle, \end{aligned} \tag{3.23}$$

$$\lambda_{00} = \left(\frac{q}{p}\right)_K \frac{\bar{A}_{00}}{A_{00}}, \quad \lambda_{+-} = \left(\frac{q}{p}\right)_K \frac{\bar{A}_{+-}}{A_{+-}}. \tag{3.24}$$

Then

$$\eta_{00} = \frac{1-\lambda_{00}}{1+\lambda_{00}}, \quad \eta_{+-} = \frac{1-\lambda_{+-}}{1+\lambda_{+-}}. \tag{3.25}$$

The η_{00} and η_{+-} parameters get contributions from CP violation in mixing ($|(q/p)|_K \neq 1$) and from the interference of decays with and without mixing ($\mathcal{I}m\lambda_{ij} \neq 0$) at $\mathcal{O}(10^{-3})$ and from CP violation in decay ($|\bar{A}_{ij}/A_{ij}| \neq 1$) at $\mathcal{O}(10^{-6})$.

There are two isospin channels in $K \to \pi\pi$ leading to final $(2\pi)_{I=0}$ and $(2\pi)_{I=2}$ states:

$$\begin{aligned} \langle \pi^0\pi^0| &= \sqrt{\frac{1}{3}}\langle(\pi\pi)_{I=0}| - \sqrt{\frac{2}{3}}\langle(\pi\pi)_{I=2}|, \\ \langle \pi^+\pi^-| &= \sqrt{\frac{2}{3}}\langle(\pi\pi)_{I=0}| + \sqrt{\frac{1}{3}}\langle(\pi\pi)_{I=2}|. \end{aligned} \tag{3.26}$$

The fact that there are two strong phases allows for CP violation in decay. The possible effects are, however, small (on top of the smallness of the relevant CP violating phases) because the final $I=0$ state is dominant (this is the $\Delta I = 1/2$ rule). Defining

$$A_I = \langle(\pi\pi)_I|\mathcal{H}|K^0\rangle, \quad \bar{A}_I = \langle(\pi\pi)_I|\mathcal{H}|\bar{K}^0\rangle, \tag{3.27}$$

we have, experimentally,

$$|A_2/A_0| \approx 1/20. \tag{3.28}$$

Instead of η_{00} and η_{+-} we may define two combinations, ε_K and ε'_K, in such a way that the possible effects of CP violation in decay (mixing) are isolated into ε'_K (ε_K).

The experimental definition of the ε_K parameter is

$$\varepsilon_K \equiv \frac{1}{3}(\eta_{00} + 2\eta_{+-}). \tag{3.29}$$

To zeroth order in A_2/A_0, we have $\eta_{00} = \eta_{+-} = \varepsilon_K$. However, the specific combination (3.29) is chosen in such a way that the following relation holds to *first* order in A_2/A_0:

$$\varepsilon_K = \frac{1-\lambda_0}{1+\lambda_0}. \tag{3.30}$$

Since, by definition, only one strong channel contributes to λ_0, there is indeed no CP violation in decay in (3.30). It is simple to show that $\mathcal{R}e\ \varepsilon_K \neq 0$ is a manifestation of CP violation in mixing while $\mathcal{I}m\ \varepsilon_K \neq 0$ is a manifestation of CP violation in the interference between decays with and without mixing. Since experimentally $\arg\varepsilon_K \approx \pi/4$, the two contributions are comparable.

The experimental definition of the ε'_K parameter is

$$\varepsilon'_K \equiv \frac{1}{3}(\eta_{+-} - \eta_{00}). \tag{3.31}$$

The theoretical expression is

$$\varepsilon'_K \approx \frac{1}{6}(\lambda_{00} - \lambda_{+-}). \tag{3.32}$$

Obviously, any type of CP violation which is independent of the final state does not contribute to ε'_K. Consequently, there is no contribution from CP violation in mixing to (3.32). It is simple to show that $\mathcal{R}e\ \varepsilon'_K \neq 0$ is a manifestation of CP violation in decay while $\mathcal{I}m\ \varepsilon'_K \neq 0$ is a manifestation of CP violation in the interference between decays with and without mixing.

3.5. CP violation in $K \to \pi\nu\bar{\nu}$

CP violation in the rare $K \to \pi\nu\bar{\nu}$ decays is very interesting. It is very different from the CP violation that has been observed in $K \to \pi\pi$ decays which is small and involves theoretical uncertainties. Similar to the CP asymmetry in $B \to \psi K_S$, it is predicted to be large and can be cleanly interpreted. Furthermore, observation of the $K_L \to \pi^0\nu\bar{\nu}$ decay at the rate predicted by the Standard Model will provide evidence that CP violation cannot be attributed to mixing ($\Delta F = 2$) processes only, as in superweak models.

Define the decay amplitudes

$$A_{\pi^0\nu\bar{\nu}} = \langle\pi^0\nu\bar{\nu}|\mathcal{H}|K^0\rangle, \quad \bar{A}_{\pi^0\nu\bar{\nu}} = \langle\pi^0\nu\bar{\nu}|\mathcal{H}|\bar{K}^0\rangle, \tag{3.33}$$

and the related $\lambda_{\pi\nu\bar{\nu}}$ quantity:

$$\lambda_{\pi\nu\bar{\nu}} = \left(\frac{q}{p}\right)_K \frac{\bar{A}_{\pi^0\nu\bar{\nu}}}{A_{\pi^0\nu\bar{\nu}}}. \tag{3.34}$$

The decay amplitudes of $K_{L,S}$ into a final $\pi^0\nu\bar{\nu}$ state are then

$$\langle\pi^0\nu\bar{\nu}|\mathcal{H}|K_{L,S}\rangle = pA_{\pi^0\nu\bar{\nu}} \mp q\bar{A}_{\pi^0\nu\bar{\nu}}, \tag{3.35}$$

and the ratio between the corresponding decay rates is

$$\frac{\Gamma(K_L \to \pi^0\nu\bar{\nu})}{\Gamma(K_S \to \pi^0\nu\bar{\nu})} = \frac{1+|\lambda_{\pi\nu\bar{\nu}}|^2 - 2\mathcal{R}e\lambda_{\pi\nu\bar{\nu}}}{1+|\lambda_{\pi\nu\bar{\nu}}|^2 + 2\mathcal{R}e\lambda_{\pi\nu\bar{\nu}}}. \tag{3.36}$$

We learn that the $K_L \to \pi^0\nu\bar{\nu}$ decay rate vanishes in the CP limit ($\lambda_{\pi\nu\bar{\nu}} = 1$), as expected on general grouns [39].

Since the effects of CP violation in decay and in mixing are expected to be negligibly small, $\lambda_{\pi\nu\bar{\nu}}$ is, to an excellent approximation, a pure phase. Defining θ_K to be the relative phase between the $K-\bar{K}$ mixing amplitude and the $s \to d\nu\bar{\nu}$ decay amplitude, namely $\lambda_{\pi\nu\bar{\nu}} = e^{2i\theta_K}$, we get from (3.36):

$$\frac{\Gamma(K_L \to \pi^0\nu\bar{\nu})}{\Gamma(K_S \to \pi^0\nu\bar{\nu})} = \frac{1-\cos 2\theta_K}{1+\cos 2\theta_K} = \tan^2\theta_K. \tag{3.37}$$

Using the isospin relation $A(K^0 \to \pi^0\nu\bar{\nu})/A(K^+ \to \pi^+\nu\bar{\nu}) = 1/\sqrt{2}$, we get

$$a_{\pi\nu\bar{\nu}} \equiv \frac{\Gamma(K_L \to \pi^0\nu\bar{\nu})}{\Gamma(K^+ \to \pi^+\nu\bar{\nu})} = \frac{1-\cos 2\theta_K}{2} = \sin^2\theta_K. \tag{3.38}$$

Note that $a_{\pi\nu\bar{\nu}} \leq 1$, and consequently a measurement of $\Gamma(K^+ \to \pi^+\nu\bar{\nu})$ can be used to set a model independent upper limit on $\Gamma(K_L \to \pi^0\nu\bar{\nu})$ [26].

3.6. CP Violation in $D \to K\pi$ Decays

Within the Standard Model, $D-\bar{D}$ mixing is expected to be well below the experimental bound. Furthermore, effects related to CP violation in $D-\bar{D}$ mixing are expected to be negligibly small since this mixing is described to a good approximation by physics of the first two generations. An experimental observation of $D-\bar{D}$ mixing close to the present bound (and, obviously, of related CP violation) will then be evidence for New

Physics. We now give the formalism of the neutral D system for the case that the mixing is close to the present bounds.

We define the neutral D mass eigenstates:

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle. \tag{3.39}$$

We define the following four decay amplitudes:

$$\begin{aligned} A_{K^+\pi^-} &= \langle K^+\pi^-|\mathcal{H}|D^0\rangle, \quad \bar{A}_{K^+\pi^-} = \langle K^+\pi^-|\mathcal{H}|\bar{D}^0\rangle, \\ A_{K^-\pi^+} &= \langle K^-\pi^+|\mathcal{H}|D^0\rangle, \quad \bar{A}_{K^-\pi^+} = \langle K^-\pi^+|\mathcal{H}|\bar{D}^0\rangle. \end{aligned} \tag{3.40}$$

We introduce the following two quantities:

$$\lambda_{K^+\pi^-} = \left(\frac{q}{p}\right)_D \frac{\bar{A}_{K^+\pi^-}}{A_{K^+\pi^-}}, \quad \lambda_{K^-\pi^+} = \left(\frac{q}{p}\right)_D \frac{\bar{A}_{K^-\pi^+}}{A_{K^-\pi^+}}. \tag{3.41}$$

The following approximations are all experimentally confirmed:

$$\Delta m_D \ll \Gamma_D; \quad \Delta\Gamma_D \ll \Gamma_D; \quad |\lambda^{-1}_{K^+\pi^-}| \ll 1; \quad |\lambda_{K^-\pi^+}| \ll 1. \tag{3.42}$$

We are interested in the case that Δm_D is found to be close to the present bound. (As mentioned above, this could only happen with new physics beyond the Standard Model.) Then, we can make a second approximation:

$$\Delta\Gamma_D \ll \Delta m_D. \tag{3.43}$$

We further make the reasonable assumptions that CP violation in decay is negligible:

$$\left|\frac{A_{K^+\pi^-}}{\bar{A}_{K^-\pi^+}}\right| = \left|\frac{\bar{A}_{K^+\pi^-}}{A_{K^-\pi^+}}\right| = 1, \tag{3.44}$$

and that CP violation in mixing is negligible:

$$\left|\left(\frac{q}{p}\right)_D\right| = 1. \tag{3.45}$$

With (3.44) and (3.45), we find

$$|\lambda^{-1}_{K^+\pi^-}| = |\lambda_{K^-\pi^+}| \equiv |\lambda_{K\pi}|. \tag{3.46}$$

For the observation of mixing, we are interested in the state $|D^0(t)\rangle$ that starts out as a pure $|D^0\rangle$ at $t=0$ and in the state $|\bar{D}^0(t)\rangle$ that starts out as a pure $|\bar{D}^0\rangle$. The result of the above discussion is the following form for the (time dependent) ratio between the doubly Cabibbo suppressed and Cabibbo allowed rates:

$$\begin{aligned}\frac{\Gamma[D^0(t)\to K^+\pi^-]}{\Gamma[D^0(t)\to K^-\pi^+]} =&|\lambda_{K\pi}|^2+\frac{(\Delta m_D)^2}{4}t^2+\mathcal{I}m(\lambda^{-1}_{K^+\pi^-})\Delta m_D t,\\ \frac{\Gamma[\bar{D}^0(t)\to K^-\pi^+]}{\Gamma[\bar{D}^0(t)\to K^+\pi^-]} =&|\lambda_{K\pi}|^2+\frac{(\Delta m_D)^2}{4}t^2+\mathcal{I}m(\lambda_{K^-\pi^+})\Delta m_D t.\end{aligned} \tag{3.47}$$

(These are approximate expressions that hold for time $t \lesssim \frac{1}{\Gamma_D}$.)

The linear term is potentially CP violating. There are four possibilities concerning this term [47,48]:

(i) $\mathcal{I}m(\lambda^{-1}_{K^+\pi^-})=\mathcal{I}m(\lambda_{K^-\pi^+})=0$: both strong and weak phases play no role in these processes.

(ii) $\mathcal{I}m(\lambda^{-1}_{K^+\pi^-})=\mathcal{I}m(\lambda_{K^-\pi^+})\neq 0$: weak phases play no role in these processes. There is a different strong phase shift in $D^0\to K^+\pi^-$ and $D^0\to K^-\pi^+$.

(iii) $\mathcal{I}m(\lambda^{-1}_{K^+\pi^-})=-\mathcal{I}m(\lambda_{K^-\pi^+})\neq 0$: strong phases play no role in these processes. CP violating phases affect the mixing amplitude.

(iv) $|\mathcal{I}m(\lambda^{-1}_{K^+\pi^-})|\neq|\mathcal{I}m(\lambda_{K^-\pi^+})|$: both strong and weak phases play a role in these processes.

4. CP Violation in The Standard Model

4.1. Introduction

The irremovable phase in the CKM matrix allows CP violation. Recalling the CP transformation laws,

$$\bar{\psi}_i\psi_j\to\bar{\psi}_j\psi_i,\quad \bar{\psi}_i\gamma^\mu W_\mu(1-\gamma_5)\psi_j\to\bar{\psi}_j\gamma^\mu W_\mu(1-\gamma_5)\psi_i, \tag{4.1}$$

we learn that mass terms and gauge interactions can be CP invariant if the masses and couplings are real. In particular, consider the coupling of $W^\pm$ to quarks. It has the form

$$gV_{ij}\bar{u}_i\gamma_\mu W^{+\mu}(1-\gamma_5)d_j+gV^*_{ij}\bar{d}_j\gamma_\mu W^{-\mu}(1-\gamma_5)u_i. \tag{4.2}$$

The CP operation interchanges the two terms except that V_{ij} and V_{ij}^* are not interchanged. Thus, CP is a good symmetry only if there is a mass basis where all couplings and masses are real.

CP is not necessarily violated in the three generation SM. If two quarks of the same charge had equal masses, one mixing angle and the phase could be removed from $V_{\rm CKM}$. Thus CP violation requires

$$(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_t^2 - m_u^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2)(m_b^2 - m_d^2) \neq 0. \tag{4.3}$$

If the value of any of the three mixing angles were 0 or $\pi/2$, then again the phase could be removed. Finally, CP would not be violated if the value of the single phase were 0 or π. These last eight conditions are elegantly incorporated into one, parameterization independent condition, that is (see (2.45) for the definition of J):

$$J \neq 0. \tag{4.4}$$

(In the parameterization (2.17) $J = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta$. This shows explicitly that $J \neq 0$ is equivalent to $\theta_{ij} \neq 0, \pi/2$ and $\delta \neq 0, \pi$.) The fourteen conditions incorporated in (4.3) and (4.4) can all be written as a single condition on the mass matrices in the interaction basis [27]:

$$\mathcal{I}m\{\det[M_d M_d^\dagger, M_u M_u^\dagger]\} \neq 0 \;\Leftrightarrow\; \text{CP violation}. \tag{4.5}$$

The quantity J is of much interest in the study of CP violation from the CKM matrix. The maximum value that J might assume is $1/(6\sqrt{3}) \approx 0.1$, but in reality it is $\sim 3\times 10^{-5}$.

Since the Standard Model contains only a single independent CP-violating phase, all possible CP-violating effects in this theory are very closely related. Consequently, the pattern of CP-violations in B decays is strongly constrained. The goal of B factories is to test whether this pattern occurs in Nature.

4.2. CP Violation in Mixing

In the B_d system we expect that $\Gamma_{12} \ll M_{12}$ model independently. Using the SM box diagrams to estimate the two quantities [49], one gets

$$\frac{\Gamma_{12}}{M_{12}} = -\frac{3\pi}{2}\frac{1}{f_2(m_t^2/m_W^2)}\frac{m_b^2}{m_t^2}\left(1 + \frac{8}{3}\frac{m_c^2}{m_b^2}\frac{V_{cb}V_{cd}^*}{V_{tb}V_{td}^*}\right). \tag{4.6}$$

This confirms our order of magnitude estimate, $|\Gamma_{12}/M_{12}| \lesssim 10^{-2}$. CP violation in mixing is proportional to $\mathcal{I}m(\Gamma_{12}/M_{12})$ which is even further suppressed:

$$1 - \left|\frac{q}{p}\right| = \frac{1}{2}\mathcal{I}m\frac{\Gamma_{12}}{M_{12}} = \frac{4\pi}{f_2(m_t^2/m_W^2)}\frac{m_c^2}{m_t^2}\frac{J}{|V_{tb}V_{td}^*|^2} \sim 10^{-3}. \tag{4.7}$$

Note that the suppression comes from the (m_c^2/m_t^2) factor. The last term is the ratio of the area of the unitarity triangle to the length of one of its sides squared, so it is $\mathcal{O}(1)$. In contrast, for the B_s system, where (4.7) holds except that V_{td} is replaced by V_{ts}, there is an additional suppression from $J/|V_{tb}V_{ts}^*|^2 \sim 10^{-2}$ (see the corresponding unitarity triangle).

The above estimate of CP violation in mixing suffers from large uncertainties (of order 30% [13] or even higher [50]) related to the use of a quark diagram to describe Γ_{12} .

4.3. CP Violation in Hadronic Decays of Neutral B

In the previous subsection we estimated the effect of CP violation in mixing to be of $\mathcal{O}(10^{-3})$ within the Standard Model, and $\leq \mathcal{O}(|\Gamma_{12}/M_{12}|) \sim 10^{-2}$ model independently [51]. In semileptonic decays, CP violation in mixing is the leading effect and therefore it can be measured through $a_{\rm SL}$. In purely hadronic B decays, however, CP violation in decay and in the interference of decays with and without mixing is $\gtrsim \mathcal{O}(10^{-2})$. We can therefore safely neglect CP violation in mixing in the following discussion and use

$$\frac{q}{p} = -\frac{M_{12}^*}{|M_{12}|} = \frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\omega_B. \tag{4.8}$$

A crucial question is then whether CP violation in decay is comparable to the CP violation in the interference of decays with and without mixing or negligible. In the first case, we can use the corresponding charged B decays to observe effects of CP violation in decay. In the latter case, CP asymmetries in neutral B decays are subject to clean theoretical interpretation: we will either have precise measurements of CKM parameters or be provided with unambiguous evidence for new physics. The question of the relative size of CP violation in decay can only be answered on a channel by channel basis, which is what we do in this section.

Most channels have contributions from both tree- and three types of penguin-diagrams, the latter classified according to the identity of the quark in the loop, as diagrams with different intermediate quarks may have both different strong phases and different weak phases [52]. On the other hand, the subdivision of tree processes into spectator, exchange and annihilation diagrams is unimportant in this respect since they all carry the same weak phase.

While quark diagrams can be easily classified in this way, the description of B decays is not so neatly divided into tree and penguin contributions once long distance physics effects are taken into account. Rescattering processes can change the quark content of the final state and confuse the identification of a contribution. There is no physical distinction between rescattered tree diagrams and long-distance contributions to the cuts of a penguin diagram. While these issues complicate estimates of various rates, they can always be avoided in describing the weak phase structure of B-decay amplitudes. The decay amplitudes for $b \to q\bar{q}q'$ can always be written as a sum of three terms with definite CKM coefficients:

$$A(q\bar{q}q') = V_{tb}V^*_{tq'}P^t_{q'} + V_{cb}V^*_{cq'}(T_{c\bar{c}q'}\delta_{qc} + P^c_{q'}) + V_{ub}V^*_{uq'}(T_{u\bar{u}q'}\delta_{qu} + P^u_{q'}). \tag{4.9}$$

Here P and T denote contributions from tree and penguin diagrams, excluding the CKM factors. As they stand, the P terms are not well defined because of the divergences of the penguin diagrams. Only differences of penguin diagrams are finite and well defined. However already we see that diagrams that can be mixed by rescattering effects always appear with the same CKM coefficients and hence that a separation of these terms is not needed when discussing weak phase structure. Now it is useful to use eqs. (2.10) and (2.11) to eliminate one of the three terms, by writing its CKM coefficient as minus the sum of the other two.

In the case of $q\bar{q}s$ decays it is convenient to remove the $V_{tb}V^*_{ts}$ term. Then

$$\begin{aligned} A(c\bar{c}s) &= V_{cb}V^*_{cs}(T_{c\bar{c}s} + P^c_s - P^t_s) + V_{ub}V^*_{us}(P^u_s - P^t_s), \\ A(u\bar{u}s) &= V_{cb}V^*_{cs}(P^c_s - P^t_s) + V_{ub}V^*_{us}(T_{u\bar{u}s} + P^u_s - P^t_s), \\ A(s\bar{s}s) &= V_{cb}V^*_{cs}(P^c_s - P^t_s) + V_{ub}V^*_{us}(P^u_s - P^t_s). \end{aligned} \tag{4.10}$$

In these expressions only differences of penguin contributions occur, which makes the cancellation of the ultraviolet divergences of these diagrams explicit. Furthermore, the second term has a CKM coefficient that is much smaller, by $\mathcal{O}(\lambda^2)$, than the first. Hence this grouping is useful in classifying the expected CP violation in decay. (Note that terms $b \to d\bar{d}s$, which have only penguin contributions, mix strongly with the $u\bar{u}s$ terms and hence cannot be separated from them. Thus P terms in $A(u\bar{u}s)$ include contributions from both $d\bar{d}s$ and $u\bar{u}s$ diagrams.)

In the case of $q\bar{q}d$ decays the three CKM coefficients are of similar magnitude. The convention is then to retain the $V_{tb}V_{td}^*$ term because, in the Standard Model, the phase difference between this weak phase and half the mixing weak phase is zero. Thus only one unknown weak phase enters the calculation of the interference between decays with and without mixing. We can choose to eliminate which of the other terms does not have a tree contribution. In the cases $q = s$ or d, since neither has a tree contribution either term can be removed. Thus we write

$$\begin{aligned} A(c\bar{c}d) &= V_{tb}V_{td}^*(P_d^t - P_d^u) + V_{cb}V_{cd}^*(T_{c\bar{c}d} + P_d^c - P_d^u), \\ A(u\bar{u}d) &= V_{tb}V_{td}^*(P_d^t - P_d^c) + V_{ub}V_{ud}^*(T_{u\bar{u}d} + P_d^u - P_d^c), \\ A(s\bar{s}d) &= V_{tb}V_{td}^*(P_d^t - P_d^u) + V_{cb}V_{cd}^*(P_d^c - P_d^u). \end{aligned} \tag{4.11}$$

Again only differences of penguin amplitudes occur. Furthermore the difference of penguin terms that occurs in the second term would vanish if the charm and up quark masses were equal, and thus is GIM suppressed [53]. However, even in modes with no tree contribution, $(s\bar{s}d)$, the interference of the two terms can still give significant CP violation in the interference of decays with and without mixing.

The penguin processes all involve the emission of a neutral boson, either a gluon (strong penguins) or a photon or Z boson (electroweak penguins). Excluding the CKM coefficients, the ratio of the contribution from the difference between a top and light quark strong penguin diagram to the contribution from a tree diagram is of order

$$r_{PT} = \frac{P^t - P^{light}}{T_{q\bar{q}q'}} \approx \frac{\alpha_s}{12\pi} \ln \frac{m_t^2}{m_b^2}. \tag{4.12}$$

This is a factor of $\mathcal{O}(0.03)$. However this estimate does not include the effect of hadronic matrix elements, which are the probability factor to produce a particular final state particle content from a particular quark content. Since this probability differs for different kinematics, color flow and spin structures, it can be different for tree and penguin contributions and may partially compensate the coupling constant suppression of the penguin term. Recent CLEO results on $BR(B \to K\pi)$ and $BR(B \to \pi\pi)$ [54] suggest that the matrix element of penguin operators is indeed enhanced compared to that of tree operators. The enhancement could be by a factor of a few, leading to

$$r_{PT} \sim \lambda^2 - \lambda. \tag{4.13}$$

(Note that r_{PT} does not depend on the CKM parameters. We use powers of the Wolfenstein parameter λ to quantify our estimate for r_{PT} is order to simplify the comparison between the size of CP violation in decay and CP violation in the interference between decays with and without mixing.) Electroweak penguin difference terms are even more suppressed since they have an $\alpha_{\rm EM}$ or α_W instead of the α_s factor in (4.12), but certain Z-contributions are enhanced by the large top quark mass and so can be non-negligible.

We thus classify B decays into four classes. Classes (i) and (ii) are expected to have relatively small CP violation in decay and hence are particularly interesting for extracting CKM parameters from interference of decays with and without mixing. In the remaining two classes, CP violation in decay could be significant and the neutral decay asymmetries cannot be cleanly interpreted in terms of CKM phases.

(i) Decays dominated by a single term: $b \to c\bar{c}s$ and $b \to s\bar{s}s$. The Standard Model cleanly predicts very small CP violation in decay: $\mathcal{O}(\lambda^4 - \lambda^3)$ for $b \to c\bar{c}s$ and $\mathcal{O}(\lambda^2)$ for $b \to s\bar{s}s$. Any observation of large CP asymmetries in charged B decays for these channels would be a clue to physics beyond the Standard Model. The corresponding neutral modes have cleanly predicted relationships between CKM parameters and the measured asymmetry from interference between decays with and without mixing. The modes $B \to \psi K$ and $B \to \phi K$ are examples of this class.

(ii) Decays with a small second term: $b \to c\bar{c}d$ and $b \to u\bar{u}d$. The expectation that penguin-only contributions are supressed compared to tree contributions suggests that these modes will have small effects of CP violation in decay, of $\mathcal{O}(\lambda^2 - \lambda)$, and an approximate prediction for the relationship between measured asymmetries in neutral decays and CKM phases can be made. Examples here are $B \to DD$ and $B \to \pi\pi$.

(iii) Decays with a suppressed tree contribution: $b \to u\bar{u}s$. The tree amplitude is suppressed by small mixing angles, $V_{ub}V_{us}$. The no-tree term may be comparable or even dominate and give large interference effects. An example is $B \to \rho K$.

(iv) Decays with no tree contribution: $b \to s\bar{s}d$. Here the interference comes from penguin contributions with different charge 2/3 quarks in the loop and gives CP violation in decay that could be as large as 10% [55,56]. An example is $B \to KK$.

Note that if the penguin enhancement is significant, then some of the decay modes listed in class (ii) might actually fit better in class (iii). For example, it is possible that $b \to u\bar{u}d$ decays have comparable contributions from tree and penguin amplitudes. On the other hand, this would also mean that some modes listed in class (iii) could be dominated by a single penguin term. For such cases an approximate relationship between measured asymmetries in neutral decays and CKM phases can be made.

4.4. CP violation in the interference between B decays with and without mixing

Let us first discuss an example of class (i), $B \to \psi K_S$. A new ingredient in the analysis is the effect of $K-\bar{K}$ mixing. For decays with a single K_S in the final state, $K-\bar{K}$ mixing is essential because $B^0 \to K^0$ and $\bar{B}^0 \to \bar{K}^0$, and interference is possible only due to $K-\bar{K}$ mixing. This adds a factor of

$$\left(\frac{p}{q}\right)_K = \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}\omega_K^* \tag{4.14}$$

into $(\bar{A}/A)$. The quark subprocess in $\bar{B}^0 \to \psi\bar{K}^0$ is $b \to c\bar{c}s$ which is dominated by the W-mediated tree diagram:

$$\frac{\bar{A}_{\psi K_S}}{A_{\psi K_S}} = \eta_{\psi K_S}\left(\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\right)\left(\frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}\right)\omega_B^*. \tag{4.15}$$

The CP-eigenvalue of the state is $\eta_{\psi K_S} = -1$. Combining (4.8) and (4.15), we find

$$\lambda(B \to \psi K_S) = -\left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)\left(\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\right)\left(\frac{V_{cd}^*V_{cb}}{V_{cd}V_{cb}^*}\right) \implies \mathcal{I}m\lambda_{\psi K_S} = \sin(2\beta). \tag{4.16}$$

The second term in (4.10) is of order $\lambda^2 r_{PT}$ for this decay and thus eq. (4.16) is clean of hadronic uncertainties to $\mathcal{O}(10^{-3})$. Consequently, this measurement can give the theoretically cleanest determination of a CKM parameter, even cleaner than the determination of $|V_{us}|$ from $K \to \pi \ell \nu$. (If BR$(K_L \to \pi \nu \bar{\nu})$ is measured, it will give a comparably clean determination of η.)

A second example of a theoretically clean mode in class (i) is $B \to \phi K_S$. The quark subprocess involves FCNC and cannot proceed via a tree level SM diagram. The leading contribution comes from penguin diagrams. The two terms in eq. (4.10) are now both differences of penguins but the second term is CKM suppressed and thus of $\mathcal{O}(\lambda^2)$ compared to the first. Thus CP violation in the decay is at most a few percent, and can be neglected in the analysis of asymmetries in this channel. The analysis is similar to the ψK_S case, and the asymmetry is proportional to $\sin(2\beta)$:

The same quark subprocesses give theoretically clean CP asymmetries also in B_s decays. These asymmetries are, however, very small since the relative phase between the mixing amplitude and the decay amplitudes (β_s defined below) is very small.

The best known example of class (ii) is $B \to \pi\pi$. The quark subprocess is $b \to u\bar{u}d$ which is dominated by the W-mediated tree diagram. Neglecting for the moment the second, pure penguin, term in eq. (4.11) we find

$$\frac{\bar{A}_{\pi\pi}}{A_{\pi\pi}} = \eta_{\pi\pi} \frac{V_{ub} V_{ud}^*}{V_{ub}^* V_{ud}} \omega_B^*. \tag{4.17}$$

The CP eigenvalue for two pions is $+1$. Combining (4.8) and (4.17), we get

$$\lambda(B \to \pi^+\pi^-) = \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right) \left(\frac{V_{ud}^* V_{ub}}{V_{ud} V_{ub}^*}\right) \implies \mathcal{I}m \lambda_{\pi\pi} = \sin(2\alpha). \tag{4.18}$$

The pure penguin term in eq. (4.11) has a weak phase, $\arg(V_{td}^* V_{tb})$, different from the term with the tree contribution, so it modifies both $\mathcal{I}m\lambda$ and (if there are non-trivial strong phases) $|\lambda|$. The recent CLEO results mentioned above suggest that the penguin contribution to $B \to \pi\pi$ channel is significant, probably 10% or more. This then introduces CP violation in decay, unless the strong phases cancel (or are zero, as suggested by factorization arguments). The resulting hadronic uncertainty can be eliminated using isospin analysis [57]. This requires a measurement of the rates for the isospin-related channels $B^+ \to \pi^+\pi^0$ and $B^0 \to \pi^0\pi^0$ as well as the corresponding CP-conjugate processes. The rate for $\pi^0\pi^0$ is expected to be small and the measurement is difficult, but even an upper bound on this rate can be used to limit the magnitude of hadronic uncertainties [58].

Related but slightly more complicated channels with the same underlying quark structure are $B \to \rho^0\pi^0$ and $B \to a_1^0\pi^0$. Again an analysis involving the isospin-related channels can be used to help eliminate hadronic uncertainties from CP violations in the decays [59,60]. Channels such as $\rho\rho$ and $a_1\rho$ could in principle also be studied, using angular analysis to determine the mixture of CP-even and CP-odd contributions.

The analysis of $B \to D^+D^-$ proceeds along very similar lines. The quark subprocess here is $b \to c\bar{c}d$, and so the tree contribution gives

$$\lambda(B \to D^+D^-) = \eta_{D^+D^-}\left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)\left(\frac{V_{cd}^*V_{cb}}{V_{cd}V_{cb}^*}\right) \implies \mathcal{I}m\lambda_{DD} = -\sin(2\beta). \qquad (4.19)$$

since $\eta_{D^+D^-} = +1$. Again, there are hadronic uncertainties due to the pure penguin term in (4.11), but they are estimated to be small.

In all cases the above discussions have neglected the distinction between strong penguins and electroweak penguins. The CKM phase structure of both types of penguins is the same. The only place where this distinction becomes important is when an isospin argument is used to remove hadronic uncertainties due to penguin contributions. These arguments are based on the fact that gluons have isospin zero, and hence strong penguin processes have definite ΔI. Photons and Z-bosons on the other hand contribute to more than one ΔI transition and hence cannot be separated from tree terms by isospin analysis. In most cases electroweak penguins are small, typically no more than ten percent of the corresponding strong penguins and so their effects can safely be neglected. However in cases (iii) and (iv), where tree contributions are small or absent, their effects may need to be considered. (A full review of the role of electroweak penguins in B decays has been given in ref. [61].)

4.5. Unitarity Triangles

One can obtain an intuitive understanding of the Standard Model CP violation in the interference between decays with and without mixing by examining the unitarity triangles. It is instructive to draw the three triangles, (2.9), (2.10) and (2.11), knowing the experimental values (within errors) for the various $|V_{ij}|$. In the first triangle (2.9), one side is of $\mathcal{O}(\lambda^5)$ and therefore much shorter than the other, $\mathcal{O}(\lambda)$, sides. In the second triangle

(2.10), one side is of $\mathcal{O}(\lambda^4)$ and therefore shorter than the other, $\mathcal{O}(\lambda^2)$, sides. In the third triangle (2.11), all sides have lengths of $\mathcal{O}(\lambda^3)$. The first two triangles then almost collapse to a line while the third one is open.

Let us examine the CP asymmetries in the leading decays into final CP eigenstates. For the B mesons, the size of these asymmetries (*e.g.* $\mathcal{I}m\lambda_{\psi K_S}$) depends on β because it gives the difference between half the phase of the $B-\bar{B}$ mixing amplitude and the phase of the decay amplitudes. The form of the third unitarity triangle, (2.11), implies that $\beta = \mathcal{O}(1)$, which explains why these asymmetries are expected to be large.

It is useful to define the analog phases for the B_s meson, β_s, and the K meson, β_K:

$$\beta_s \equiv \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right], \quad \beta_K \equiv \arg\left[-\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*}\right]. \tag{4.20}$$

The angles β_s and β_K can be seen to be the small angles of the second and first unitarity triangles, (2.10) and (2.9), respectively. This gives an intuitive understanding of why CP violation is small in the leading K decays (that is ε_K measured in $K\to\pi\pi$ decays) and is expected to be small in the leading B_s decays (*e.g.* $B_s\to\psi\phi$). Decays related to the short sides of these triangles are rare but could exhibit significant CP violation. Actually, the large angles in the (2.9) triangle are approximately β and $\pi-\beta$, which explains why CP violation in $K\to\pi\nu\bar{\nu}$ is related to β and expected to be large. The large angles in the (2.10) triangle are approximately γ and $\pi-\gamma$. This explains why the CP asymmetry in $B_s\to\rho K_S$ is related to γ and expected to be large. (Note, however, that this mode gets comparable contributions from penguin and tree diagrams and does not give a clean CKM measurement [56].)

5. CP Violation Beyond the Standard Model

The Standard Model picture of CP violation is rather unique and highly predictive. In particular, we would like to point out the following features:

(i) CP is broken explicitly.

(ii) All CP violation arises from a single phase, that is $\delta_{\rm KM}$.

(iii) The measured value of ε_K requires that $\delta_{\rm KM}$ is of order one. (In other words, CP is not an approximate symmetry of the Standard Model.)

(iv) The values of all other CP violating observables can be predicted. In particular, CP violation in $B \to \psi K_S$ (and similarly various other CP asymmetries in B decays), and in $K \to \pi\nu\bar{\nu}$ are expected to be of order one.

The commonly repeated statement that CP violation is one of the least tested aspects of the Standard Model is well demonstrated by the fact that none of the above features necessarily holds in the presence of New Physics. In particular, there are viable models of new physics (*e.g.* certain supersymmetric models) with the following features:

(i) CP is broken spontaneously.

(ii) There are many CP violating phases (even in the low energy effective theory).

(iii) CP is an approximate symmetry, with all CP violating phases small (usually $10^{-3} \lesssim \phi_{\rm CP} \lesssim 10^{-2}$).

(iv) Values of CP violating observables can be predicted and could be very different from the Standard Model predictions (except, of course, ε_K). In particular, $\mathcal{I}m\lambda_{\psi K_S}$ and $a_{\pi\nu\bar{\nu}}$ could both be $\ll 1$.

To understand how the Standard Model predictions could be modified by New Physics, we will focus on CP violation in the interference between decays with and without mixing. As explained above, it is this type of CP violation which, due to its theoretical cleanliness, may give unambiguous evidence for New Physics most easily.

5.1. CP Violation as a Probe of Flavor Beyond the Standard Model

Let us consider five specific CP violating observables.

(i) $\mathcal{I}m\lambda_{\psi K_S}$, the CP asymmetry in $B \to \psi K_S$. This measurement will cleanly determine the relative phase between the $B-\bar{B}$ mixing amplitude and the $b \to c\bar{c}s$ decay amplitude ($\sin 2\beta$ in the Standard Model). The $b \to c\bar{c}s$ decay has Standard Model tree contributions and therefore is very unlikely to be significantly affected by new physics. On the other hand, the mixing amplitude can be easily modified by new physics. We parametrize such a modification by a phase θ_d:

$$\mathcal{I}m\lambda_{\psi K_S} = \sin[2(\beta+\theta_d)]. \tag{5.1}$$

(ii) $\mathcal{I}m\lambda_{\phi K_S}$, the CP asymmetry in $B \to \phi K_S$. This measurement will cleanly determine the relative phase between the $B-\bar{B}$ mixing amplitude and the $b \to s\bar{s}s$ decay amplitude. The $b \to s\bar{s}s$ decay has only Standard Model penguin contributions and therefore is sensitive to new physics. We parametrize the modification of the decay amplitude by a phase θ_A [62]:

$$\mathcal{I}m\lambda_{\phi K_S} = \sin[2(\beta + \theta_d + \theta_A)]. \tag{5.2}$$

(iii) $a_{\pi\nu\bar{\nu}}$, the CP violating ratio of $K \to \pi\nu\bar{\nu}$ decays. This measurement will cleanly determine the relative phase between the $K - \bar{K}$ mixing amplitude and the $s \to d\nu\bar{\nu}$ decay amplitude. The experimentally measured small value of ε_K requires that the phase of the $K - \bar{K}$ mixing amplitude is not modified from the Standard Model prediction. On the other hand, the decay, which in the Standard Model is a loop process with small mixing angles, can be easily modified by new physics.

(iv) $\mathcal{I}m(\lambda_{K^-\pi^+})$, the CP violating quantity in $D \to K^-\pi^+$ decay. The ratio

$$a_{D\to K\pi} = \frac{\mathcal{I}m(\lambda_{K^-\pi^+})}{|\lambda_{K^-\pi^+}|} \tag{5.3}$$

depends on the relative phase between the $D - \bar{D}$ mixing amplitude and the $c \to d\bar{s}u$ decay amplitude. Within the Standard Model, this decay channel is tree level. It is unlikely that it is affected by new physics. On the other hand, the mixing amplitude can be easily modified by new physics.

(v) d_N, the electric dipole moment of the neutron. We did not discuss this quantity so far because, unlike CP violation in meson decays, flavor changing couplings are not necessary for d_N. In other words, the CP violation that induces d_N is *flavor diagonal*. It does in general get contributions from flavor changing physics, but it could be induced by sectors that are flavor blind. Within the Standard Model (and ignoring the strong CP angle $\theta_{\rm QCD}$), the contribution from $\delta_{\rm KM}$ arises at the three loop level and is at least six orders of magnitude below the experimental bound [10] $d_N^{\rm exp}$,

$$d_N^{\rm exp} = 1.1 \times 10^{-25}\ e\ \text{cm}. \tag{5.4}$$

The various CP violating observables discussed above are sensitive then to new physics in the mixing amplitudes for the $B - \bar{B}$ and $D - \bar{D}$ systems, in the decay amplitudes for $b \to s\bar{s}s$ and $s \to d\nu\bar{\nu}$ channels and to flavor diagonal CP violation. If information about all these processes becomes available and deviations from the Standard Model predictions are found, we can ask rather detailed questions about the nature of the new physics that is responsible to these deviations:

(i) Is the new physics related to the down sector? the up sector? both?

(ii) Is the new physics related to $\Delta B = 1$ processes? $\Delta B = 2$? both?

(iii) Is the new physics related to the third generation? to all generations?

(iv) Are the new sources of CP violation flavor changing? flavor diagonal? both?

It is no wonder then that with such rich information, flavor and CP violation provide an excellent probe of new physics.

5.2. Supersymmetry

A generic supersymmetric extension of the Standard Model contains a host of new flavor and CP violating parameters. (For reviews on supersymmetry see refs. [63-66]. The following section is based on [67].) The requirement of consistency with experimental data provides strong constraints on many of these parameters. For this reason, the physics of flavor and CP violation has had a profound impact on supersymmetric model building. A discussion of CP violation in this context can hardly avoid addressing the flavor problem itself. Indeed, many of the supersymmetric models that we analyze below were originally aimed at solving flavor problems.

As concerns CP violation, one can distinguish two classes of experimental constraints. First, bounds on nuclear and atomic electric dipole moments determine what is usually called the *supersymmetric CP problem*. Second, the physics of neutral mesons and, most importantly, the small experimental value of ε_K pose the *supersymmetric ε_K problem*. The latter is closely related to the flavor structure of supersymmetry.

The contribution to the CP violating ε_K parameter in the neutral K system is dominated by diagrams involving Q and $\bar{d}$ squarks in the same loop [68-72]. The corresponding effective four-fermi operator involves fermions of both chiralities, so that its matrix elements are enhanced by $\mathcal{O}(m_K/m_s)^2$ compared to the chirality conserving operators. For $m_{\tilde{g}} \simeq m_Q \simeq m_D = \tilde{m}$ (our results depend only weakly on this assumption) and focusing on the contribution from the first two squark families, one gets (we use the results in ref.

[72])

$$\frac{(\Delta m_K \varepsilon_K)^{\rm SUSY}}{\Delta m_K \varepsilon_K} \sim 10^7 \left(\frac{300\ {\rm GeV}}{\tilde m}\right)^2 \left|\frac{(\delta m^2_D)_{12}}{\tilde m^2}\right|^2 |K^d_{12}|^2 \sin\phi, \tag{5.5}$$

where $\tilde m$ is the typical scale of squark and gluino masses, $(\delta m^2_D)_{12}$ is the mass-squared difference between the first two down squark generations, K^d_{12} is the mixing angle in the gluino-quark-squark coupling and ϕ is the relevant CP violating phase in the mixing. In a generic supersymmetric framework, we expect $\tilde m = \mathcal{O}(m_Z)$, $\delta m^2_D/\tilde m^2 = \mathcal{O}(1)$, $K^d_{ij} = \mathcal{O}(1)$ and $\sin\phi = \mathcal{O}(1)$. Then the constraint (5.5) is generically violated by about seven orders of magnitude. Eq. (5.5) also shows what are the possible solutions to the supersymmetric flavor and CP problems:

(i) *Universality*: At some high scale, the soft supersymmetry breaking terms are universal. In other words, the different squark generations are degenerate [73-74]. There are two very different ways to achieve such a situation. First, the mechanism that communicates supersymmetry breaking to the observable sector could be flavor blind. This is the case with gauge mediated supersymmetry breaking [75-78], but it is also possible (though not generic) that similar boundary conditions occur when supersymmetry breaking is communicated to the observable sector up at the Planck scale [79-85]. RGE effects will introduce some splitting at low energy which, for the first two squark generations, is typically of $\mathcal{O}(m_c^2/m_W^2)$. Second, the Yukawa hierarchy could be a result of a non-Abelian flavor symmetry with the first two generations forming a doublet [86-94]. In this framework, the first two squark generations are approximately degenerate with splitting which could be as high as $\mathcal{O}(\lambda^2)$. The third generation could be widely split from the first two.

(ii) *Alignment* [95-97]: The mixing angles in the gluino-quark-squark couplings are small. This is usually achieved in models where the Yukawa hierarchy is explained by Abelian flavor symmetries. In the symmetry limit, both the quark mass matrices and the squark mass-squared matrices are diagonal, so that mixing is suppressed by small breaking parameters. Typically, the alignment is required to be very precise between the first two down generations, while all other supersymmetric mixing angles are similar to the corresponding CKM angles.

(iii) *Heavy Squarks* [86,89,98-100]: If the masses of the first and second generation squarks m_i are larger than the other soft masses, $m_i^2 \sim 100\,\tilde{m}^2$, then the Supersymmetric CP problem is solved and the ε_K problem is relaxed (but not eliminated). This does not necessarily lead to naturalness problems, since these two generations are almost decoupled from the Higgs sector.

(iv) *Approximate CP* [101-103].: Both supersymmetric CP problems are solved if CP is an approximate symmetry, broken by a small parameter of order 10^{-3}. Of course, some mechanism to solve the supersymmetric flavor problems has to be invoked.

Measurements of CP violation will provide us with an excellent probe of the flavor and CP structure of supersymmetry. This is clearly demonstrated in the Table below.

Model	$d_N/d_N^{\rm exp}$	θ_d	θ_A	$a_{D\to K\pi}$	$a_{K\to\pi\nu\bar\nu}$
Standard Model	$\lesssim 10^{-6}$	0	0	0	$\mathcal{O}(1)$
Exact Universality	$\lesssim 10^{-6}$	0	0	0	=SM
Approximate CP	$\sim 10^{-1}$	$-\beta$	0	$\mathcal{O}(10^{-3})$	$\mathcal{O}(10^{-5})$
Alignment	$\gtrsim 10^{-3}$	$\mathcal{O}(0.2)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	≈SM
Approx. Universality	$\gtrsim 10^{-2}$	$\mathcal{O}(0.2)$	$\mathcal{O}(1)$	0	≈SM
Heavy Squarks	$\sim 10^{-1}$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(10^{-2})$	≈SM

CP violating observables in various classes of Supersymmetric flavor models.

5.3. Final Comments

The unique features of CP violation are well demonstrated by examining the CP asymmetry in $B \to \psi K_S$, $\mathcal{I}m\lambda_{\psi K_S}$, and CP violation in $K \to \pi\nu\bar\nu$, $\mathcal{I}m\lambda_{\pi\nu\bar\nu}$. Model independently, $\mathcal{I}m\lambda_{\psi K_S}$ measures the relative phase between the $B-\bar B$ mixing amplitude and the $b \to c\bar c d$ decay amplitude (more precisely, the $b \to c\bar c s$ decay amplitude times the $K-\bar K$ mixing amplitude), while $\mathcal{I}m\lambda_{\pi\nu\bar\nu}$ measures the relative phase between the $K-\bar K$ mixing amplitude and the $s \to d\nu\bar\nu$ decay amplitude. We would like to emphasize the following three points:

(i) *The two measurements are theoretically clean to better than* $\mathcal{O}(10^{-2})$. Thus they can provide the most accurate determination of CKM parameters.

(ii) *As concerns CP violation, the Standard Model is a uniquely predictive model.* In particular, it predicts that the seemingly unrelated $\mathcal{I}m\lambda_{\psi K_S}$ and $\mathcal{I}m\lambda_{\pi\nu\bar{\nu}}$ measure the same parameter, that is the angle β of the unitarity triangle.

(iii) *In the presence of New Physics, there is in general no reason for a relation between* $\mathcal{I}m\lambda_{\psi K_S}$ *and* $\mathcal{I}m\lambda_{\pi\nu\bar{\nu}}$. Therefore, a measurement of both will provide a sensitive probe of New Physics.

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