Appendix A1 in ZLF

Variational Derivatives

Without giving strict justification, we shall explain here several simple rules of calculating variational derivatives. They follow from the fact that $\delta/\delta f(r)$ generalizes the notion of partial derivative $\partial/\partial f(r_n)$ (r_n is discrete) for the case of continual number of variables.

1. The variational derivatives of linear functional of the form $I = \int \phi(r')f(r')\,dr'$ are calculated by the simple formula

$$\frac{\delta I}{\delta f(r)} = \int \phi(r')\frac{\delta f(r')}{\delta f(r)}\,dr' = \int \phi(r')\delta(r-r')\,dr' = \phi(r)\ . \qquad (A1.1)$$

To obtain this formula, one can mentally substitute $\delta/\delta f(r)$ by $\partial/\partial f(r_n)$, simultaneously replacing integration by summation, and after differentiation, return to continuous form of record:

$$\frac{\delta}{\delta f(r)}\int \phi(r')f(r')\,dr' \to \frac{\partial}{\partial f(r_n)}\sum_m \phi(r_m)f(r_m) = \phi(r_n) \to \phi(r)\ .$$

This result may be represented by a symbolic formula:

$$\frac{\delta f(r')}{\delta f(r)} = \delta(r-r')\ . \qquad (A1.2)$$

2. If the f function in the functional is affected by differential operators, then, in order to make use of the rule (A1.2), one should at first “throw them over” to the left, fulfilling integration by parts. For exaple,

$$\frac{\delta}{\delta f(r)}\int \phi\nabla f\,dr' = -\frac{\delta}{\delta f(r)}\int f\nabla\phi\,dr' = -\nabla\phi\ . \qquad (A1.3)$$

We assumed here that on the boundary of integration domain the product $f(r')\phi(r')$ becomes zero.

The variational derivative of nonlinear functionals is calculated according to the rule of differentiating a complex function similarly to partial derivatives:

$$\frac{\delta}{\delta f(r)}\int F[f(r')]\,dr' = \int \frac{\delta F}{\delta f(r')}\frac{\delta f(r')}{\delta f(r)}\,dr' = \frac{\delta F}{\delta f(r)}\ .$$

For example,

$$\frac{\delta}{\delta f(r)} \int f^n(r')\, dr' = n f^{n-1}(r), \quad \text{etc .} \tag{A1.4}$$

3. Variation of multidimensional integrals over the function giving the boundary of integration domain is not so trivial. Thus, in deriving the Hamiltonian description of waves on a fluid surface (see Sect. 1.2, ZLF), one should calculate the variational derivative $\delta/\delta\eta(r)$ of the following functional

$$J = \int d\vec{r} \int^{\eta(\vec{r})} A[\vec{r}, z; \eta(r)]\, dz. \tag{A1.5}$$

Here $\vec{r} = (x, y)$ is a two-dimensional vector, $A[\vec{r}, z, \eta(\vec{r})]$ depends not only on spatial variables but also on the form of the $\eta(\vec{r})$ function. For example, a boundary condition on A is given on the surface $z = \eta(\vec{r})$. With variation $\eta \to \eta + \delta\eta$, the J variation then consists of two terms

$$\delta J = \int d\vec{r}'\, A[\vec{r}', z; \eta(\vec{r}')]_{z=\eta} \delta\eta(\vec{r}') + \int d\vec{r}' \delta A[\vec{r}', z; \eta(r')]_{z=\eta}. \tag{A1.6}$$

The first term is due to variation in the size of integration domain; the second one, to variation of the integrand function, e.g. $\delta A = A(z + \delta z) - A(z) = \delta\eta \partial A/\partial z$ [see (1.2.33)].

Appendix A2 in ZLF

Canonicity Condition of Transformations

1. Let $a_j(r,t)$, $a_j^*(r,t)$ be the canonical variables, so that their equations of motion have a canonical form (1.1.6). Let us introduce new variables $b_l(r,t)$, $b_l^*(r,t)$ using the transformations which do not contain time in an explicit form

$$b_l = f_l\{a_j, a_j^*\}, \quad b_l^* = f_l^*\{a_j, a_j^*\}. \qquad (A2.1)$$

Here f_i is a certain functional. Let us obtain the conditions under which the equations of motion in the b, b^* variables retain the form (1.1.6):

$$\frac{\partial b_l(r,t)}{\partial t} = \sum_j \int dr' \left[-i \frac{\delta b_l(r)}{\delta a_j(r')} \frac{\delta \mathcal{H}}{\delta a_j * (r')} + i \frac{\delta b_l(r)}{\delta a_j^*(r')} \frac{\delta \mathcal{H}}{\delta a_j(r')} \right],$$

$$\frac{\delta \mathcal{H}}{\delta a_j(r')} = \sum_m \int dr'' \left[\frac{\delta \mathcal{H}}{\delta b_m(r'')} \frac{\delta b_m(r'')}{\delta a_j(r')} + \frac{\delta \mathcal{H}}{\delta b_m^*(r'')} \frac{\delta b_m^*(r'')}{\delta a_j(r')} \right],$$

$$i\frac{\partial b_l}{\partial t} = \sum_j \int dr' \left[\frac{\delta \mathcal{H}}{\delta b_m^*(r')} \{b_l(r) b_m^*(r')\}_{aa^*} + \frac{\delta \mathcal{H}}{\delta b_m(r')} \{b_l(r) b_m(r')\}_{aa^*} \right]. \qquad (A2.2)$$

We have introduced here designations for the Poisson brackets

$$\{f(r)g(r')\}_{aa^*} = \sum_m \int dr'' \left[\frac{\delta f(r)}{\delta a_m(r'')} \frac{\delta g(r')}{\delta a_m^*(r'')} - \frac{\delta f(r)}{\delta a_m^*(r'')} \frac{\delta g(r')}{\delta a_m(r'')} \right]. \qquad (A2.3)$$

For the equations (A2.2) to have a canonical form (1.1.6), the following conditions should be satisfied

$$\{b_l, b_j\} = 0, \quad \{b_l(r) b_j^*(r')\} = \delta_{lj}\delta(r - r')\ , \qquad (A2.4)$$

which are classical analogues of commutation equations for Bose operators.

The canonicity conditions for transformations of Fourier images $b(k,t)$ and $b^*(k,t)$ have the same form. Thus, for the linear $u - v$ transformation (1.1.16) diagonalizing the quadratic part of the Hamiltonian

$$b_j(k) = \sum_l [u_{jl}(k) a_l(k) + v_{jl}(k) a_l^*(-k)],$$

we obtain from (A2.4) the following canonicity conditions

$$\sum_l [u_{jl}(k)u^*_{ml}(k) - v_{jl}(k)v^*_{ml}(k)] = \delta_{jm},$$

$$\sum_l [u_{jl}(k)v_{ml}(-k) - v_{jl}(k)u_{ml}(-k)] = 0. \qquad (A2.5)$$

For quasi-linear transformations of the type

$$b(k) = a(k) + \int [A(k,k_1,k_2)a(k_1)a(k_2) + B(k,k_1,k_2)a^*(k_1)a(k_2)$$

$$+C(k,k_1,k_2)a^*(k_1)a^*(k_2)]\, dk_1 dk_2 \ ,$$

used in Sects. 1.1,2 to eliminate the nonresonance three-wave processes, the canonicity conditions are also obtained using the Poisson brackets (A2.4) with an accuracy to next order terms and have the form [see also next Appendix to Lecture]:

$$B(k,k_1,k_2) = B(k_1,k,k_2) = -2A^*(k_2,k_1,k) \ ,$$

$$C(k,k_1,k_2) = C(k_1,k,k_2) = C(k,k_2,k_1) \ . \qquad (A2.6)$$

2. The canonical transformations may be formulated in terms of generating functionals, which are the continual analogues of the generating functions of finite-dimensionality systems. As known, the Hamilton equations of motion may be obtained from the extremum action principle represented in the form

$$\delta \int dt \left[\int p(r,t) \frac{\partial q(r,t)}{\partial t}\, dr - \mathcal{H}\{p,q\} \right] = 0. \qquad (A2.7)$$

In this equation, all coordinates and momenta should be independently varied. For the equation in new variables P and Q also to have a canonical form, a similar principle should be satisfied

$$\delta \int dt \left[\int P(r,t) \frac{\partial Q(r,t)}{\partial t}\, dr - \mathcal{H}\{P,Q\} \right] = 0. \qquad (A2.8)$$

These two principles will be equivalent on condition that the sub-integrands differ by a total variation (analog of total differential) of the arbitrary functional Φ of coordinates, momenta and time

$$\delta\Phi = \int p(r)\delta q(r)\, dr - \int P(r)\delta Q(r)\, dr + (\mathcal{H}' - \mathcal{H})dt. \qquad (A2.9)$$

Hence we obtain the equations

$$p = \frac{\delta\Phi}{\delta q}, \quad P = -\frac{\delta\Phi}{\delta Q}, \quad \mathcal{H}' = \mathcal{H} + \frac{\partial\Phi}{\partial t}, \tag{A2.10}$$

specifying the relation (at a given $\Phi\{q, Q, t\}$) between the old and new variables, and the new Hamiltonian.

In Sect. 1.2.4, we needed the generating functional in the Q, p variables. To derive the transformation formulas in this case, one should fulfil in (A2.9) the Legendre transformation

$$\delta\Big(\Phi + \int q(r)p(r)\,dr\Big) = \int q(r)\delta p(r)\,dr$$
$$- \int P(r)\delta Q(r)\,dr + (\mathcal{H}' - \mathcal{H})\,dt\ . \tag{A2.11}$$

The new generating functional is thus equal to $F(p, Q, t) = \Phi + pq$, and

$$q = \frac{\delta F}{\delta p}, \quad P = -\frac{\delta F}{\delta Q}, \quad \mathcal{H}' = \mathcal{H} + \frac{\partial F}{\partial t}\ . \tag{A2.12}$$

It should be noted that usually it is exactly satisfiability of the condition (A2.9) or (A2.11) that is adopted as a definition of the canonicity of the $(p, q) \to (P, Q)$ transformation. Though the canonical form of equations of motion is retained by transformations of a wider class, for example, those in which the Hamiltonian is multiplied off by an arbitrary constant.

Appendix A3 in ZLF

Elimination of Non-resonant Terms from the Interaction Hamiltonian

We shall show here how one can make in the nondecay case a transformation eliminating three-wave and nonresonant four-wave processes from the interaction Hamiltonian. Let us seek the transformation as a power series. Since expansion of the Hamiltonian starts with a quadratic term and ends (for us) with fourth-order terms, the transformation should contain linear, quadratic and cubic terms.

Let us demonstrate a simple method for computing the coefficients of a power series transformation, which is canonical due to its derivation. That method is based on the fact that a hamiltonian system possesses hamiltonian properties at any time. Therefore, the transformation $c(k,0) \to c(k,t)$ is canonical. Let us consider an auxiliary Hamiltonian in the standard form (1.1.24)

$$\begin{aligned}\tilde{\mathcal{H}} &= \frac{1}{2}\int \delta(\vec{k}_1 - \vec{k}_2 - \vec{k}_3)(\tilde{V}_{123} c_1^* c_2 c_3 + \text{c.c.})\, d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 \\ &+ \frac{1}{6}\int \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)(\tilde{U}_{123} c_1^* c_2^* c_3^* + \text{c.c})\, d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 \\ &+ \frac{1}{4}\int \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4)\tilde{W}_{1234} c_1^* c_2^* c_3 c_4\, d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 d\vec{k}_4 \\ &+ \int \delta(\vec{k}_1 - \vec{k}_2 - \vec{k}_3 - \vec{k}_4)(\tilde{G}_{1234} c_1 c_2^* c_3^* c_4^* + \text{c.c.})\, d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 d\vec{k}_4 \\ &+ \int \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)(\tilde{R}_{1234} c_1^* c_2^* c_3^* c_4^* + \text{c.c.})\, d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 d\vec{k}_4. \end{aligned} \tag{A3.1}$$

Here c.c. means complex conjugation.

We can express old variables $b(k,t) = c(k,t)$ in terms of $c(k,0)$ using a Taylor series as follows

$$b(k,t) = c(k,0) + t\left(\frac{\partial c(k,t)}{\partial t}\right)_{t=0} + \frac{t^2}{2}\left(\frac{\partial^2 c(k,t)}{\partial t^2}\right)_{t=0} + \dots\ . \tag{A3.2}$$

According to (A3.1):

$$\left(\frac{\partial c(k,t)}{\partial t}\right)_{t=0} = -i\frac{\delta\mathcal{H}\{c(k,0),c^*(k,0)\}}{\delta c^*(k,0)}\ ,$$

$$\left(\frac{\partial^2 c(k,t)}{\partial t^2}\right)_{t=0} = -i\frac{\partial}{\partial t}\frac{\delta\mathcal{H}}{\delta c^*}\ . \qquad (A3.3)$$

Substituting for $\partial c/\partial t$ and $\partial^2 c/\partial t^2$ and setting, for example, $t=1$ (other choice of t just corresponds to the redefinition of transformation coefficients) we get a general view of the canonical transformations in the form of a power series:

$$b(k) = c(k) - \frac{i}{2}\int[\tilde{V}_{k12}\delta(\vec{k}-\vec{k}_1-\vec{k}_2)c_1c_2$$
$$+2\tilde{V}^*_{1k2}\delta(\vec{k}_1-\vec{k}-\vec{k}_2)c_1c_2^* + \tilde{U}_{k12}\delta(\vec{k}+\vec{k}_1+\vec{k}_2)c_1^*c_2^*]\,d\vec{k}_1d\vec{k}_2$$
$$+\int\Big[\delta(\vec{k}-\vec{k}_1-\vec{k}_2-\vec{k}_3)c_1c_2c_3(-\tilde{G}^*_{k123}-\frac{1}{4}\tilde{V}_{k1k-1}\tilde{V}_{2+323}$$
$$+\frac{1}{4}\tilde{V}^*_{1k1-k}\tilde{U}_{-2-323}) + \delta(\vec{k}+\vec{k}_1+\vec{k}_2-\vec{k}_3)c_1^*c_2^*c_3(-3i\tilde{G}_{3k12}$$
$$+\frac{1}{4}\tilde{V}_{k3k-3}\tilde{U}_{-2-121} + \frac{1}{4}\tilde{V}^*_{3k3-k}\tilde{V}^*_{2+323} - \frac{1}{2}\tilde{V}^*_{323-2}\tilde{V}^*_{1k1-k}$$
$$+\frac{1}{2}\tilde{V}_{131-3}\tilde{U}_{-k-2k2}) + \delta(\vec{k}+\vec{k}_1-\vec{k}_2-\vec{k}_3)c_1^*c_2c_3(-\frac{i}{2}\tilde{W}_{k123}$$
$$-\frac{1}{2}\tilde{V}_{k2k-2}\tilde{V}^*_{313-1} - \frac{1}{4}\tilde{V}^*_{1+k1k}\tilde{V}_{2+323} + \frac{1}{2}\tilde{V}_{131-3}\tilde{V}^*_{2k2-k}$$
$$+\frac{1}{2}\tilde{U}_{-k-1k1}\tilde{U}_{-2-323}) + \delta(\vec{k}+\vec{k}_1+\vec{k}_2+\vec{k}_3)c_1^*c_2^*c_3^*(-4i\tilde{R}_{k123}$$
$$-\frac{1}{4}\tilde{V}^*_{1+k1k}\tilde{U}_{-2-323} + \frac{1}{4}\tilde{V}^*_{2+323}\tilde{U}_{-k-1k1})\Big]\,d\vec{k}_1d\vec{k}_2d\vec{k}_3\ . \qquad (A3.4)$$

Here and below we use shorthand notation to replace the k_j arguments by the j indices.

As one cal see, that transformation consists of seven different terms but it contains only five arbitrary functions:

$$\tilde{V}(k,k_1,k_2),\ \tilde{U}(k,k_1,k_2),\ \tilde{G}(k,k_1,k_2,k_3),\ \tilde{R}(k,k_1,k_2,k_3)\,,\tilde{W}(k,k_1,k_2,k_3)\ .$$

In addition, those functions have to satisfy the usual symmetry conditions (1.1.25) for Hamiltonian coefficients. Canonicity conditions (A2.6) is satisfied identically.

Let us now substitute (A3.4) into the Hamiltonian (1.1.24). The resulting Hamiltonian will have the same form (1.1.24) but with some new coefficients. Demanding the coefficients of the cubic terms to be equal to zero, we obtain

$$i\tilde{V}_{k12} = \frac{V_{k12}}{\omega_k - \omega_1 - \omega_2}, \qquad i\tilde{U}_{k12} = -\frac{U_{k12}}{2(\omega_k + \omega_1 + \omega_2)}. \tag{A3.5}$$

They are exactly the coefficients ($2A_1 = -i\tilde{V}$, $A_3 = -i\tilde{U}$) given in (1.1.28b) for the case $\omega_k = \omega_1 = \omega_2 = \omega_3 = \omega$.

The fact that the fourth-order terms with $c_1c_2c_3c_k$ and $c_k^*c_1c_2c_3$ are equal to zero allows one to obtain another two transformation coefficients. After proper symmetrization, we find

$$\begin{aligned} i\tilde{R}_{k123} &= \frac{R_{k123} + \frac{i}{24}(U_{-k-iki}\tilde{V}_{j+ljl} + V_{k+iki}\tilde{U}_{-j-ljl})}{\omega_k + \omega_1 + \omega_2 + \omega_3} \\ &\quad + \frac{1}{48}(\tilde{V}_{j+ljl}\tilde{U}_{-k-iki} - \tilde{V}_{k+iki}\tilde{U}_{-j-ljl}), \\ i\tilde{G}_{k123} &= \frac{G_{k123} + \frac{i}{6}(V_{kik-i}\tilde{V}_{j+ljl} + V_{iki-k}\tilde{U}_{-l-jlj})}{\omega_k - \omega_1 - \omega_2 - \omega_3} \\ &\quad + \frac{1}{12}(\tilde{V}^*_{iki-k}\tilde{U}_{ljl-j} - \tilde{V}_{kik-i}\tilde{V}_{l+jlj}) \ . \end{aligned} \tag{A3.6}$$

Here summation over the divergent values of i, j, l indices running over the numbers $1, 2, 3$ is implied.

Thus, since the denominators in (A3.5–6) don't turn into zero, then respective terms may be excluded from the Hamiltonian and correspondent transformation coefficients may be obtained. The rest of the interaction Hamiltonian has the following form in the new variables

$$\begin{aligned} \mathcal{H}_{int} = \mathcal{H}_4 = \frac{1}{4}\int \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4)c_1^*c_2^*c_3c_4[W_{1234} + T_{1234} \\ +(\omega_1 + \omega_2 - \omega_3 - \omega_4)(i\tilde{W}_{1234} + \frac{1}{4}\tilde{V}^*_{pip-i}\tilde{V}_{jqj-q} \\ -\frac{1}{4}\tilde{V}^*_{qjq-j}\tilde{V}_{ipi-p})]\, d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 d\vec{k}_4 \ , \end{aligned} \tag{A3.7}$$

where

$$2T_{1234} = -\frac{U_{-3-434}U_{-1-212}}{\omega_3 + \omega_4 + \omega_{3+4}} - \frac{U_{-3-434}U_{-1-212}}{\omega_1 + \omega_2 + \omega_{1+2}}$$

$$-\frac{V^*_{1+212}V_{3+434}}{\omega_{1+2}-\omega_1-\omega_2}-\frac{V^*_{1+212}V_{3+434}}{\omega_{3+4}-\omega_3-\omega_4}-\frac{V^*_{ipi-p}V_{qjq-j}}{\omega_{q-j}+\omega_j-\omega_q}$$

and indices i, j run over the (divergent) numbers $1, 2$, indices p, q over the $3, 4$. The renormalized interaction coefficient satisfies the same symmetry conditions (1.1.25) as W_{1234} and $\tilde{W}_{1234}$.

Since we always can find such $\vec{k}_1, \ldots, \vec{k}_4$ that

$$\vec{k}_1+\vec{k}_2=\vec{k}_3+\vec{k}_4 \quad \text{and} \quad \omega_1+\omega_2=\omega_3+\omega_4, \tag{A3.8}$$

then it is impossible to eliminate the term (A3.7) in all $\vec{k}$-space. That Hamiltonian describes scattering processes allowed for all wave systems.

For consideration of weak turbulence of wave interaction with wave vectors lying only on the resonance surface (A3.8) [where (A3.7) coincides with (1.1.29)], the formula (1.1.29b) is sufficient. But if essentially nonlinear phenomena are discussed (for example, when using truncated equations for description of water waves [A.1]), the use of (1.1.29b) at $\omega_1+\omega_2 \neq \omega_3+\omega_4$ leads to the loss of energy in the equations and other errors [A.2]. The arbitrary $\tilde{W}$ function may be chosen from considerations of convenience, one must satisfy only the symmetry conditions (1.1.25). By varying $\tilde{W}$ we simultaneously vary c_k, leaving b_k constant.

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Chapter 0

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