Universal Conductivity at the Quantum Hall Liquid to Insulator Transition

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We have systematically studied the electron transport in the vicinity of the transitions from the $\nu = 1$ and $1/3$ quantum Hall liquids to the Hall insulator, in a wide variety of samples. Our results indicate that the diagonal resistivity at the transition is universal and close to the quantum unit of resistance e^2/h.

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The neighborhood of a zero temperature phase transition in two-dimensional electron systems (2DES's) is often characterized by critical behavior of transport properties. Such behavior has been observed in 2DES at high magnetic fields (B) [1,2] and in thin films near the superconductor to insulator transition [3]. Wei, Tsui, and Pruisken [1] investigated the critical behavior at the transition regions separating adjacent integer quantum Hall effect (IQHE) plateaus in semiconductor heterostructures and found that their width (ΔB) increases with temperature (T) and follows $\Delta B \sim T^{\kappa}$, with κ independent of the Landau level and sample. Subsequently, Engel *et al.* [2] showed that this universality is not restricted to the IQHE. They obtained a similar value of κ for the transition region separating the $\nu = 2/5$ and $1/3$ fractional quantum Hall effect (FQHE) states, where $\nu = n \times h/eB$ is the Landau level filling factor and n is the electron density.

Two recent discoveries shifted the attention from the inter–quantum Hall effect (QHE), plateau to plateau transitions to the transitions between the QHE incompressible liquid states and an insulating phase. The first was the observation, by Jiang *et al.* [4], of a reentrant transition to an insulating phase near the $\nu = 1/5$ FQHE liquid in the highest quality samples available. The possibility that this insulator was the electron solid expected in these nearly ideal samples generated a large amount of effort aimed at understanding the nature and origin of this unique insulating phase [5]. The second discovery was made on samples that are in the opposite extreme of the quality range in which the QHE can be observed [6–8]. These samples of very strongly disordered 2DES are insulating at $B = 0$ and undergo a B driven transition to the $\nu = 2$ IQHE state. At still higher B, they become insulating again and, surprisingly, the $\nu = 1$ state is absent. This new class of samples was investigated from a more general viewpoint, concentrating on the interplay and transitions between the various phases, rather than on their specific properties.

The work presented here bridges the gap between these two extreme cases. The samples investigated in this study exhibit metallic behavior at $B = 0$, followed by a well-developed set of QHE liquid states which terminates at high B with a single transition to an insulating phase. We thus avoid the complications of extremely high and low mobility samples and are able to address the well-defined problem of a direct, B field driven transition from a fully developed QHE liquid to an insulating phase.

Theoretically, much effort has been directed towards the establishment of a unified framework to describe the interplay between the various phases of 2DES at high B [9–11]. Kivelson, Lee, and Zhang (KLZ) [10] considered both electron correlation and disorder, present in real physical systems, and proposed a global phase diagram in the disorder-B-field plane. They applied a gauge transformation that maps the 2DES at high B to a bosonic system under a different effective B that enabled them to develop a set of correspondence rules, relating the properties of all QHE states to those of the $\nu = 1$ IQHE state. The global phase diagram is obtained by applying these rules and using the scaling results for single electron localization in high B obtained from field-theoretic studies [12] and numerical simulations [13]. The similarity we mentioned earlier of the values of κ obtained for the transitions in the FQHE and IQHE regimes is taken as evidence for their correspondence rules.

In the inset of Fig. 1(a) we depict portions of the KLZ phase diagram in the disorder–B-field plane. The solid lines represent the boundaries between the various phases. In the high disorder and high B field limits is an insulating phase, bordered by the $\nu = 1$ IQHE and the $\nu = 1/3$ and $\nu = 1/5$ FQHE liquid phases, which they named the Hall insulator (HI), because its Hall resistance has the classical value $\rho_{xy} \approx B/ne$, while its diagonal resistivity ρ_{xx} diverges as $T \to 0$. It is apparent from the figure that the topology of this phase diagram restricts the allowed continuous transitions into the HI to be only from the principal QHE states $\nu = 1, 1/3, 1/5$, etc. Experimentally, a transition to an insulating phase from the $\nu = 2/9$ FQHE liquid in an ultrahigh quality sample has been observed [14,15]. This apparent inconsistency with the KLZ phase diagram has been attributed by the

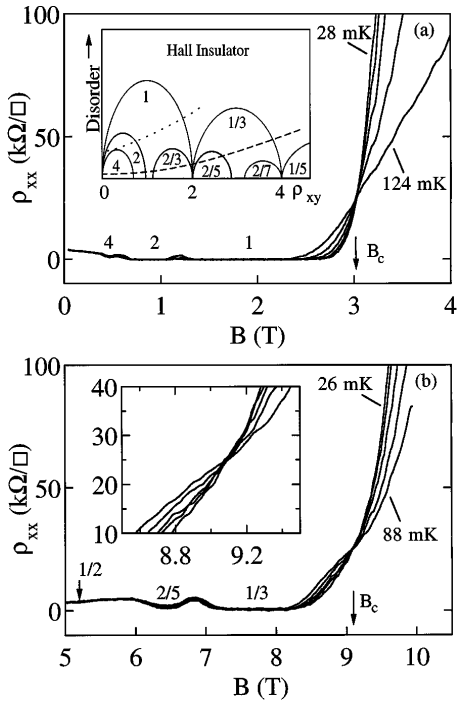

FIG. 1. ρ_{xx} vs B traces at various T's for sample 60A/B (a) and M124U2/B (b). The T's are for (a) 28, 38, 52, 77, 124 mK and (b) 26, 36, 48, 65, 88 mK. In the inset of (a) is a portion of the KLZ phase diagram (see text) in the disorder–B-field plane. The B field strength is given by ρ_{xx} in the units of h/e^2. The solid lines are phase boundaries, and the numbers are the QHE state. The dotted (dashed) line is a possible trajectory for the sample in (a) [(b)]. The inset of (b) shows the crossing point on a larger scale.

authors to the possible formation, near $\nu = 1/5$, of a Wigner-crystal-like ordered phase neglected by KLZ.

In this Letter we report on a systematic study of the critical behavior near QHE liquid to insulator transitions, from the $\nu = 1$ and $1/3$ QHE liquids, which are at sufficiently large ν, that complications due to the Wigner crystal phase are not expected. We have clearly identified a critical magnetic field B_c that marks the boundary between the QHE and the insulating phases in our GaAs/AlGaAs heterostructures. This distinct B_c exists for both low mobility [$\mu \approx (1 - 5) \times 10^4$ cm^2/V sec] samples, for which the FQHE is not resolved down to our lowest attainable T (20 mK), and high mobility ($\mu \approx 5 \times 10^5$ cm^2/V sec) samples, which exhibit a well-developed set of FQHE states and for which the transition to the insulating phase occurs from the $\nu = 1/3$ FQHE state. Our main result is that, at B_c, ρ_{xx} is independent of sample parameters and of whether the transition takes place from the $\nu = 1$ or $\nu = 1/3$ QHE state. Its value, obtained by averaging over all our samples and runs, is $\rho_{xx} = 25.3 \pm 4.1$ kΩ. The quantum unit of resistance $h/e^2 = 25.813$ kΩ is within the error of this result .

In Fig. 1, we plot a set of B field traces of ρ_{xx} taken at various T's, for both a low mobility [$\mu = 3 \times 10^4$ cm^2/V sec, Fig. 1(a)] sample and a high mobility [$\mu = 5.5 \times 10^5$ cm^2/V sec, Fig. 1(b)] sample. In the traces of each sample, a set of QHE liquid states is clearly observed. For the sample in Fig. 1(a), the $\nu = 1$ and 2 IQHE states are well developed with ρ_{xx} vanishingly small over a wide range of B. A dip near $\nu = 4$ is also seen in the traces, although it is not as well developed. No FQHE states are observed for this sample. In contrast, for the high μ sample in Fig. 1(b), the FQHE is observed. In addition to the $\nu = 1/3$ state which is fully developed, a strong dip is also apparent at $\nu = 2/5$, indicating the formation of a FQHE liquid there as well. For this sample we have chosen to depict only part of the B range above 5 T (near $\nu = 1/2$). This serves to illustrate the remarkable similarity in the general appearance of the traces from the two samples. In particular, one can identify the similarity of $\nu = 1$ and 2 states in Fig. 1(a) to the $\nu = 1/3$ and $2/5$, respectively, in Fig. 1(b). This similarity between the IQHE states at low B and the FQHE states at high B ranges in a single sample has been recently considered in the context of composite fermions [16]. Here, it may be regarded as an experimental manifestation of the law of corresponding states suggested by KLZ.

It is clear from Fig. 1 that this similarity is also seen in the transition to an insulating phase in the high B limit and that an unambiguous determination of the critical B_c for these transitions is evident. In both samples, B_c sharply separates two distinct B field regions, differing by the T dependence of ρ_{xx} in each region. This distinction can be readily seen in Fig. 2 where we plot ρ_{xx} as a function of T for several different B fields in the neighborhood of B_c (3.02 T), taken from the sample in Fig. 1(a). For the $B > B_c$ region a behavior characteristic of an insulator is observed in the sense that ρ_{xx} diverges as $T \to 0$. The behavior in the $B < B_c$ region, on the other hand, is typical of a QHE liquid, with $\rho_{xx} \to 0$ as $T \to 0$. The data are consistent with the existence of a critical point at B_c where $\rho_{xx} \to$ const as $T \to 0$, and with the scaling behavior expected near a continuous phase transition. (A detailed analysis of the scaling behavior near this transition will be presented elsewhere.) We thus identify B_c with the critical point of the QHE liquid to HI transition. We also note that the overall appearance of the data in Fig. 2 is greatly reminiscent of the superconductor to insulator transition in thin films [3].

The unique role of B_c can also be identified in the current-voltage characteristics near the transition. In Fig. 3(a) we plot a set of the four-terminal I-V curves, from the sample of Fig. 1(b), at several values of B near B_c. The curves, taken at $T = 21$ mK, are all symmetric upon reversal of the current and voltage direction and are not hysteretic. The first trace in Fig. 3(a) was taken at $B = 8.7$ T, and each consecutive trace represents an increase of 0.1 T with the last trace at 9.6 T. The I-V taken at B_c (= 9.1 T), as indicated by the dashed line, separates all traces into two groups, each showing a different type of behavior. To see this more clearly, we plot, in Fig. 1(b), the numerical derivative of the I-V

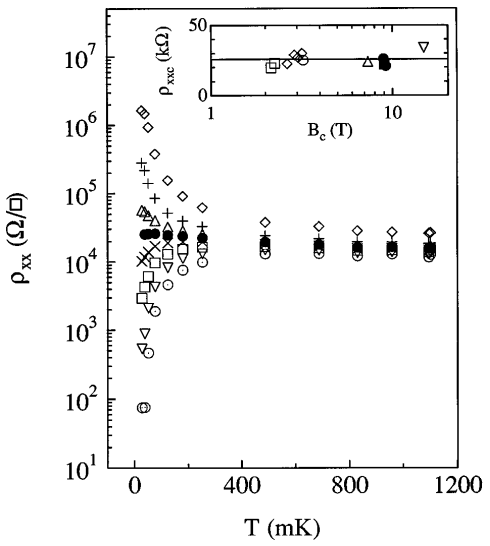

FIG. 2. ρ_{xx} vs T for $B = 4.5$ ($\diamond$), 3.5 (+), 3.2 ($\triangle$), 3.02 ($\bullet$), 2.9 ($\times$), 2.8 ($\square$), 2.7 ($\triangledown$), 2.6 ($\odot$) T for the sample in Fig. 1(a). The inset depicts ρ_{xx} vs B_c for sample 70A ($\square$), 60A ($\diamond$), 60E ($\circ$), MM051 ($\triangledown$), M124U2 ($\bullet$), the sample from Ref. [18] ($\triangle$) and the sample from Ref. [19] ($\blacksquare$). Empty (filled) symbols are for transitions from the $\nu = 1$ IQHE ($\nu = 1/3$ FQHE) state. Note that the similar symbols in the figure and in the inset are not related.

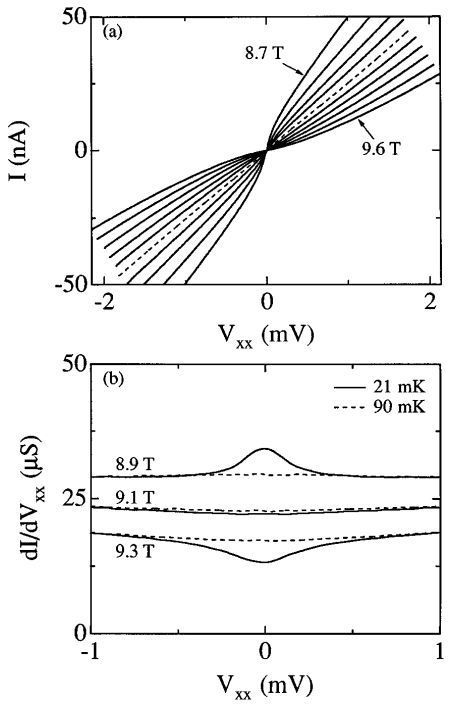

FIG. 3. (a) Four-terminal I-V traces at 0.1 T steps from $B = 8.7$ to 9.6 T for the sample in Fig. 1(b). The dashed trace is at $B = 9.1$ T. The sample is 1×0.5 mm between voltage contacts. (b) Numerical dI/dV_{xx} vs V_{xx} taken from the traces in (a).

as a function of bias at $T = 21$ and 90 mK, taken at three different B's: one in the 1/3 FQHE regime ($B = 8.9$ T), one in the HI ($B = 9.3$ T), and the one taken at B_c. At $T = 90$ mK (dashed lines) all three curves exhibit negligible (<5%) nonlinearity and are, in fact, rather similar. This is no longer the case at $T = 21$ mK (solid lines). Much stronger (20%–30%) nonlinearity develops for the $B = 8.9$ and 9.3 T curves, with a higher conductance at lower biases for the 8.9 T curve, as seen in the QHE regime in high current studies [17], and an opposite low bias behavior for the 9.3 T trace, expected for an insulator. Remarkably, the $B = B_c$ trace shows no change within experimental accuracy.

The question of whether a universal value of ρ_{xx} exists for the QHE to HI transition naturally arises. To properly answer this question, we tested a wide variety of samples and summarized our results together with the sample parameters in Table I. Our samples were grown in three different MBE machines, and were all wet etched in the shape of a Hall bar. They span a wide range of electron mobility and density, $\mu = 1.0 \times 10^4 - 5.5 \times 10^5$ cm^2/V sec and $n = (0.35 - 2.26) \times 10^{11}$ cm^{-2}, respectively. Both four-terminal ac phase sensitive technique or dc measurements were used, giving the same results. We have also included in Table I results from other researchers who observed the transition from the $\nu = 1$ IQHE [18] and the $\nu = 1/3$ FQHE [19] states.

In the inset of Fig. 2 we plot ρ_{xxc}, the value of ρ_{xx} at B_c, vs B_c for the data presented in Table I. Different cooldowns of the same physical sample have been given the same symbol with empty and full symbols denoting transitions from the $\nu = 1$ IQHE and $\nu = 1/3$ FQHE liquids, respectively. Despite the large diversity of μ, B_c, and n in our samples (cf. Table I), ρ_{xxc} is rather constant and close to h/e^2 (solid line in the inset of Fig. 2). It is clear that ρ_{xxc} is independent of any sample parameter and, remarkably, also independent of whether the transition to the HI is from the $\nu = 1$ IQHE or from the $\nu = 1/3$ FQHE liquid. We consider these results as evidence for the existence of a universal ρ_{xx} value at the QHE to HI transition.

KLZ argue a universal value for the conductivity tensor at the critical point of the transitions. To compare this prediction to our experimental results, we convert the conductivity tensor to ρ_{xx} and, surprisingly, $\rho_{xxc} = h/e^2$ for all transitions occurring from the principal QHE liquid states. This theoretical value is in good agreement with our experimental result [20].

One can see in the inset of Fig. 2 that the scatter among the ρ_{xxc} obtained for different cooldowns of a given sample is of a similar magnitude as the scatter among the different samples. This variation between cooldowns is a common feature to semiconductor heterostructures. It stems from the delicate nature of the distribution of the remote ionized donors that contribute the 2D electrons in the samples. Although the fraction of donors that are ionized remains relatively unchanged from run to run, as reflected by the small change in n, the actual donors that are ionized can vary drastically, resulting in a completely different impurity configuration and a different realization of the disordered potential seen by the conduction electrons. Consequently, different cooldowns

TABLE I. Sample parameters and results. Each state designates a separate cooldown. The last three entries are for transitions from the $\nu = 1/3$ FQHE liquid to the HI. The units of n and μ are 10^{11} cm^{-2} and 10^3 cm^2/V sec, respectively.

Sample/state	n	μ	B_c (T)	R_c (h/e^2)	ν_c
60A/B	0.42	42	3.02	1.05	0.56
60A/D	0.44	40	2.63	0.87	0.67
60A/E	0.48	52	3.17	1.16	0.61
60A/EA	0.44	30	2.89	1.12	0.61
60E/A	0.47	44	3.23	0.97	0.58
70A/B	0.35	27	2.15	0.77	0.65
70A/C	0.35	19	2.25	0.89	0.62
MM051/B	2.26	12	14.8	1.32	0.61
Ref. [18]	1.0	24	7.3	0.93	0.55
M124U2/A	0.63	500	8.88	1.01	0.28
M124U2/B	0.65	550	9.15	0.81	0.28
Ref. [19]	0.61	200	9	0.85	0.27

of the same physical sample should be regarded as different samples, albeit from the same wafer.

A universal value is expected upon averaging many realizations of the disorder as in Ref. [13]. Experimentally, this averaging may be achieved if $L \gg L_\phi$, where L is the sample size (0.5–2 mm for our samples) and L_ϕ is the electron phase coherence length. The variation of ρ_{xxc} we observe ($\sim$20%) is larger than expected for an average of $(L/L_\phi)^2$ samples even if we assume for our samples the largest value of L_ϕ reported ($\approx$0.06 mm [21]). It is not clear whether the reason for the enhanced fluctuations is extrinsic or is due to ineffective self-averaging near the QHE to HI transition [22].

In the phase diagram sketched in the inset of Figs. 1(a) and 1(b), we have plotted a possible trajectory in the disorder–B-field plane that our samples might follow (dashed lines). Regardless of the actual trajectory that the samples do follow, the transition point to the HI, ν_c, is always at $\nu > 1/2$ (1/4) for transitions from the $\nu = 1$ IQHE ($\nu = 1/3$ FQHE) liquid. This is a consequence of the expected "floating up" of the extended states due to disorder [10,23,24]. The experimental values of ν_c, given in the last column of Table I, are indeed all larger than 1/2 (1/4).

To summarize, we observed clear transitions from the $\nu = 1$ IQHE and $\nu = 1/3$ FQHE liquids to the HI and investigated the critical behavior of these transitions reflected in the $\rho_{xx}(T)$ and the current-voltage characteristics. We find that the value of ρ_{xx} at the critical point is close to h/e^2, independent of sample parameters and of whether the transition is from the $\nu = 1$ IQHE or the $\nu = 1/3$ FQHE. This result is in agreement with the existence of a universal value at the QHE to HI transition predicted by KLZ.

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