



***TIDAL DISRUPTION OF MAIN
SEQUENCE STARS INDUCED BY
MASSIVE BLACK HOLE BINARIES***

Alberto Sesana

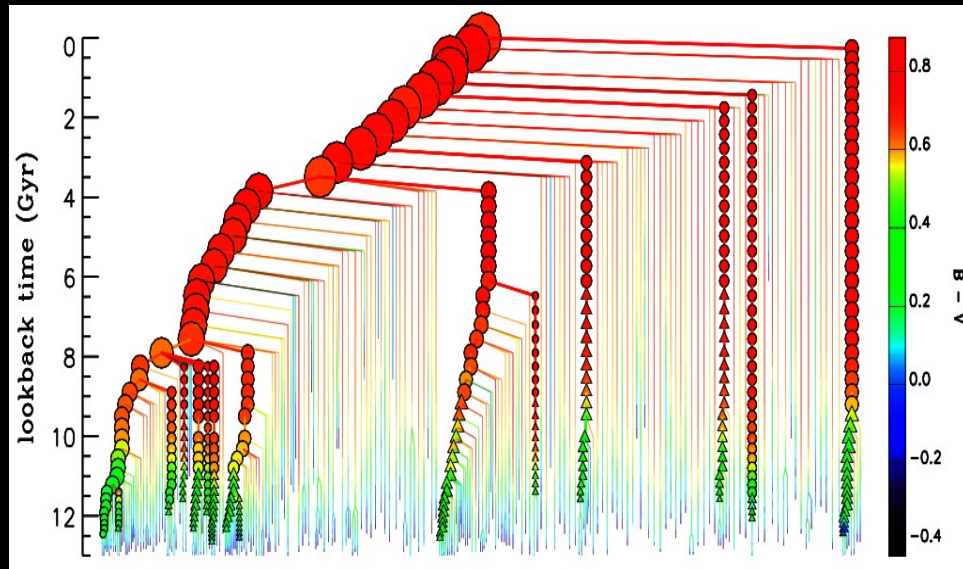
X. Chen, P. Madau, F. Liu, F. Haardt

Rehovot, 08/12/2009

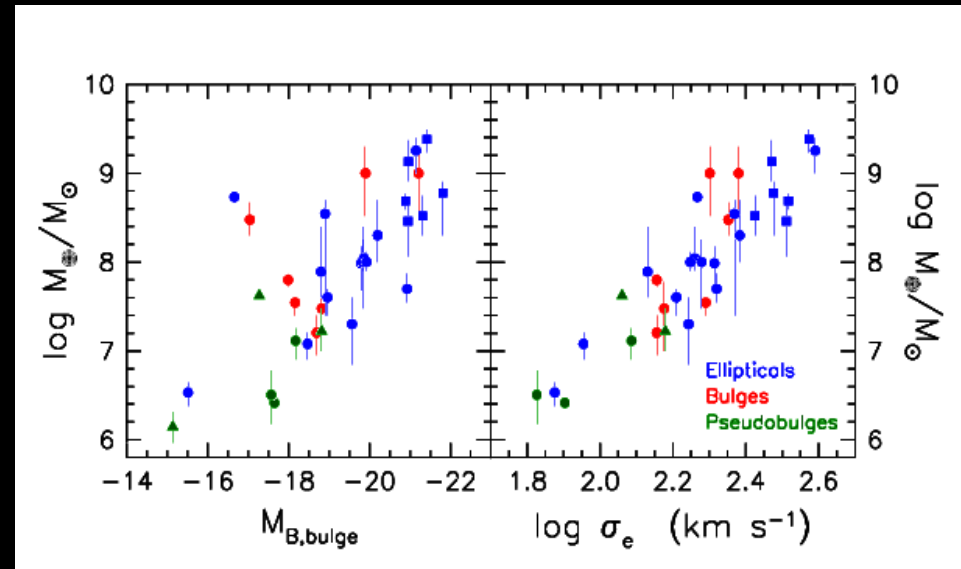
OUTLINE

- > *The essential ingredients: MBHBs and dense stellar cusps***
- > *Three body scattering experiments***
- > *Hybrid model: enhanced tidal disruption rates of main sequence stars***
- > *Don't mess with GR: Kozai vs Einstein***
- > *Numbers and observational relevance***

Structure formation in a nutshell

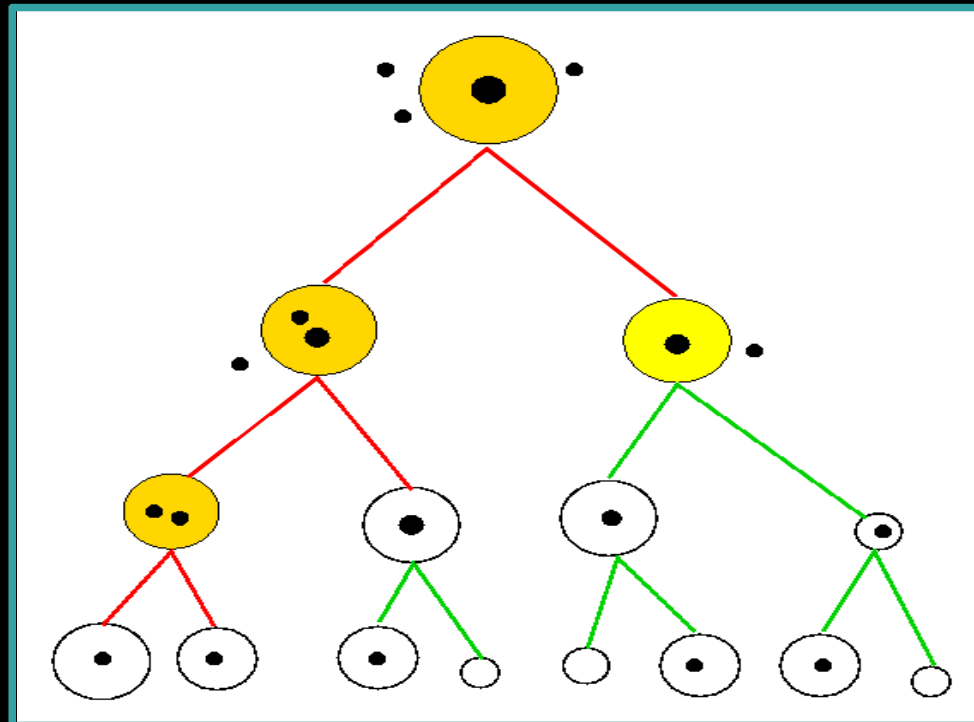


From De Lucia et al 2006



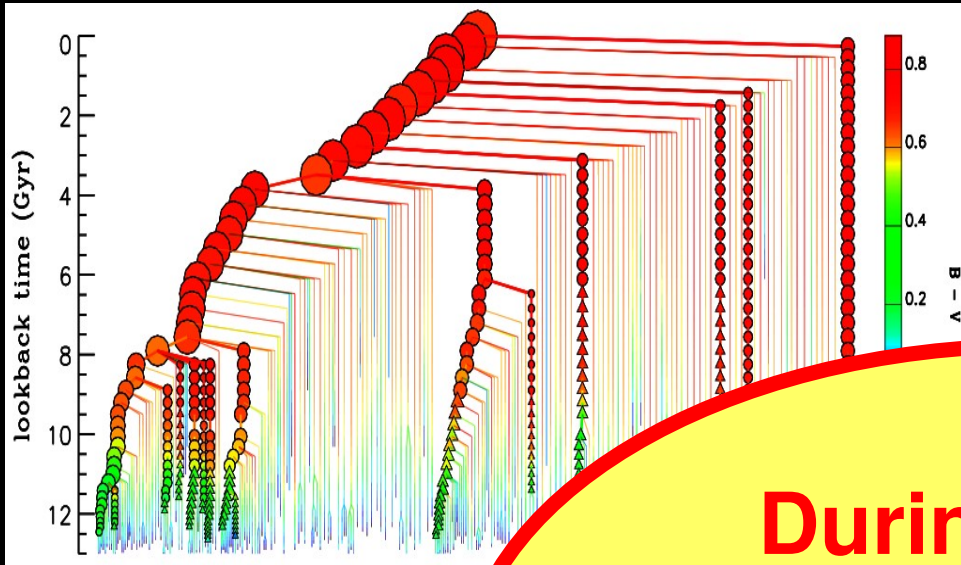
Ferrarese & Merritt 2000, Gebhardt et al. 2000

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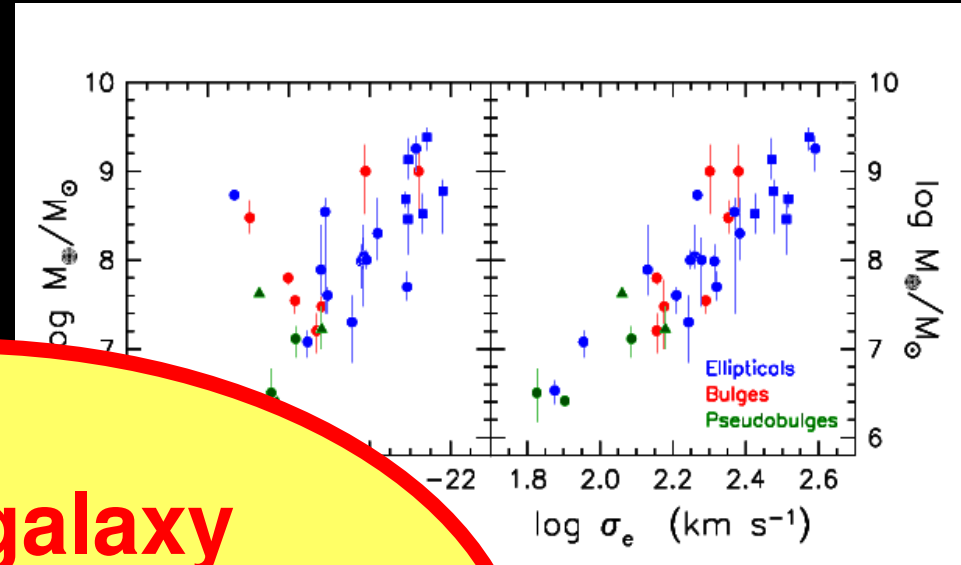


Volonteri Haardt & Madau 2003

Structure formation in a nutshell



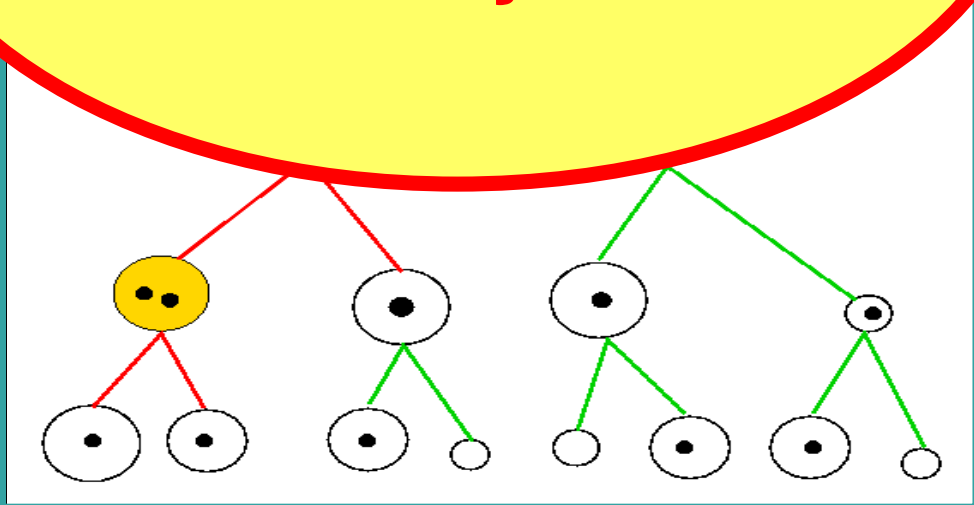
From De Lucia et al 2004



Gebhardt et al. 2000

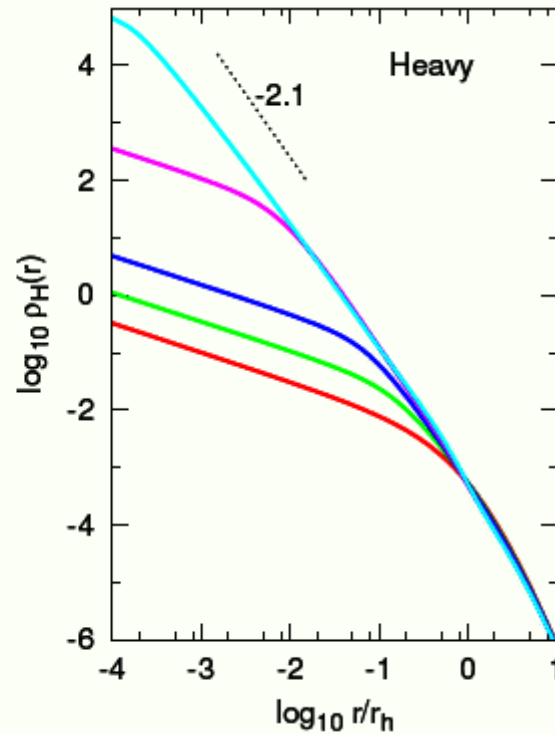
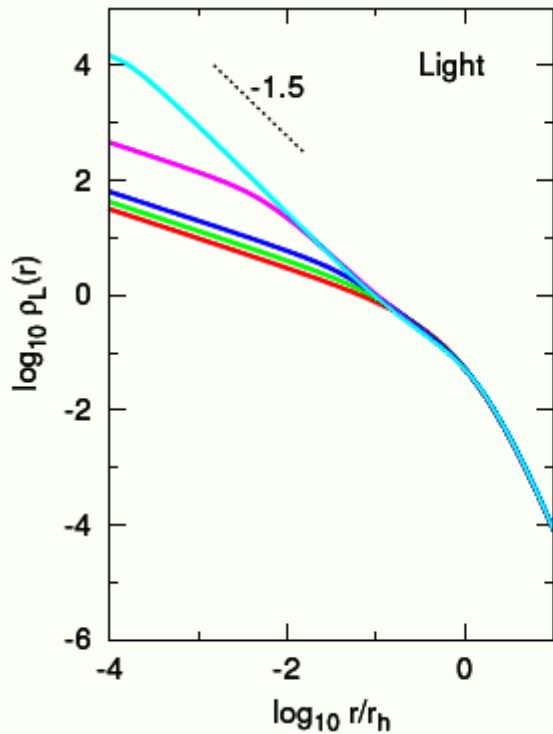
During galaxy mergers, MBHBs will inevitably form!

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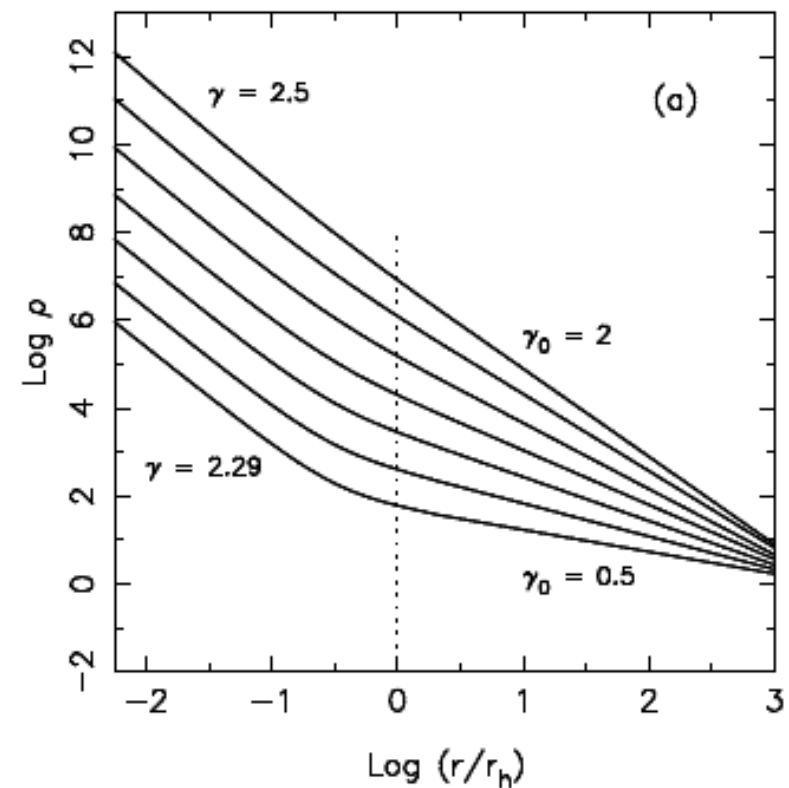


Volonteri Haardt & Madau 2003

Stellar cusps: theory



The growth of a massive object in a stellar core produces a steep power-law cusp $\rho \propto r^{-\gamma}$ with $1.5 < \gamma < 2.5$ (Young 1980)



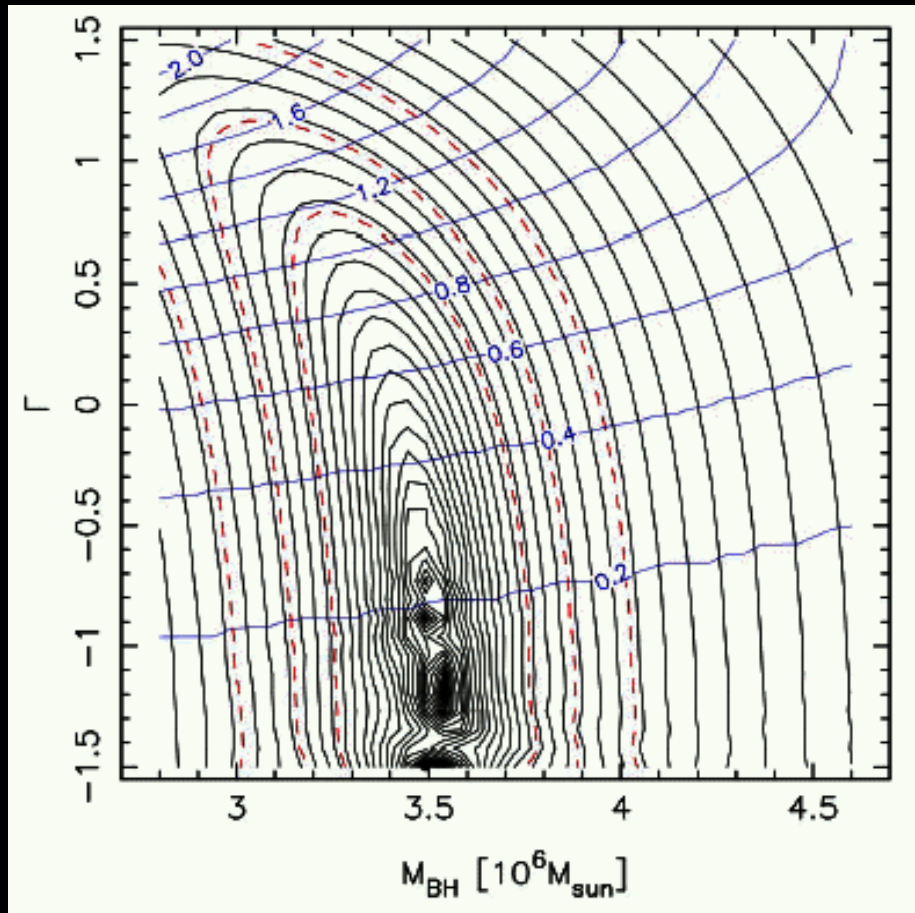
A collisional system around a massive object has a power-law steady state solution $\rho \propto r^{-7/4}$ (Bahcall & Wolf 1977)

Strong mass segregation cause the more massive objects to form even steeper cusps (Alexander & Hopman 2009)

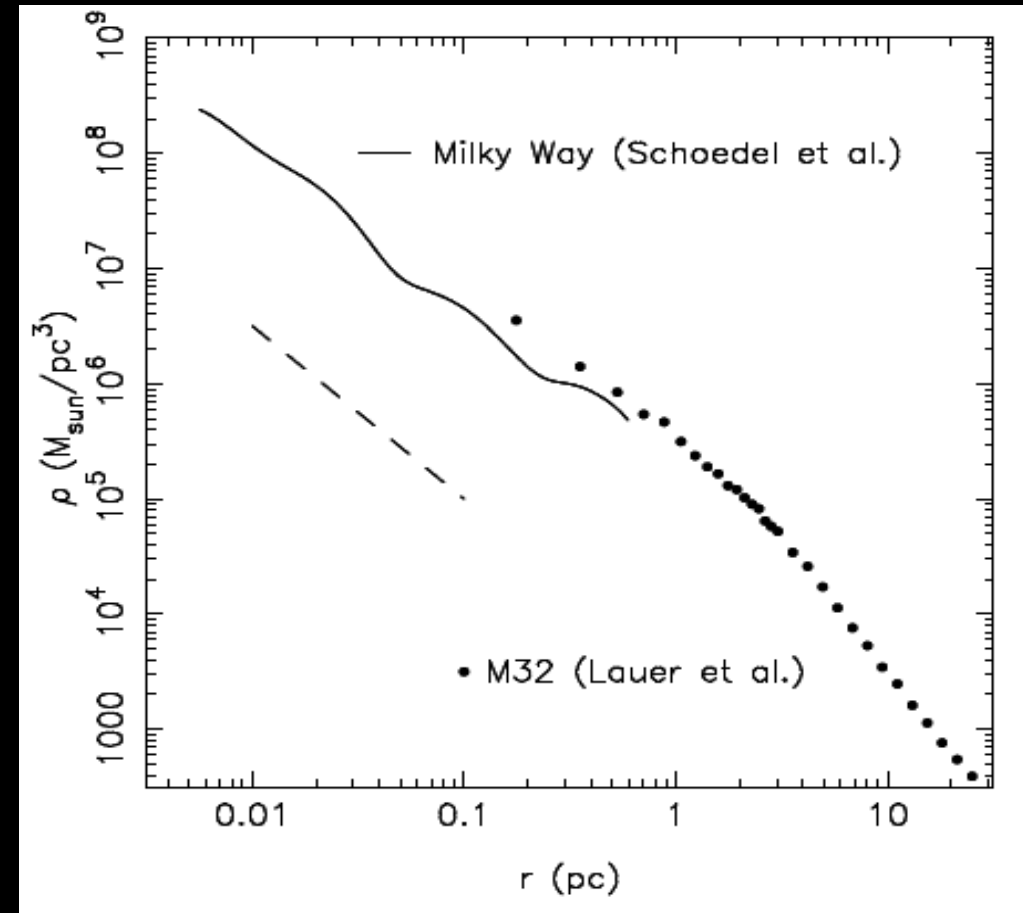
Stellar cusps: observations

Collisional nuclei ($T_{\text{rel}} < T_{\text{Hubble}}$) are expected to host steep cusps in their centres. *Should be common in spirals and in systems with $M_{\text{BH}} < 10^7 M_{\odot}$*

Difficult to detect. Lack of resolution. MBH sphere of influence resolved for a handful of galaxies out to the Virgo cluster. But *cores seem to be common.*



MW: presence of a cusp now into question (Schoedel et al. 2009)



M32 shows a power law profile down to the resolution limit (Lauer et al. 1995)

MBHB formation in a nutshell

1. dynamical friction (Lacey & Cole 1993, Colpi et al. 2000)

- from the interaction between the DM halos to the formation of the BH binary
- determined by the global distribution of matter
- efficient only for *major mergers* against mass stripping

2. binary hardening (Quinlan 1996, Milosavljevic & Merritt 2001)

- *3 bodies interactions* between the binary and the surrounding stars
- the binding energy of the BHs is larger than the thermal energy of the stars
- the SMBHs create a *stellar density core ejecting the background stars*

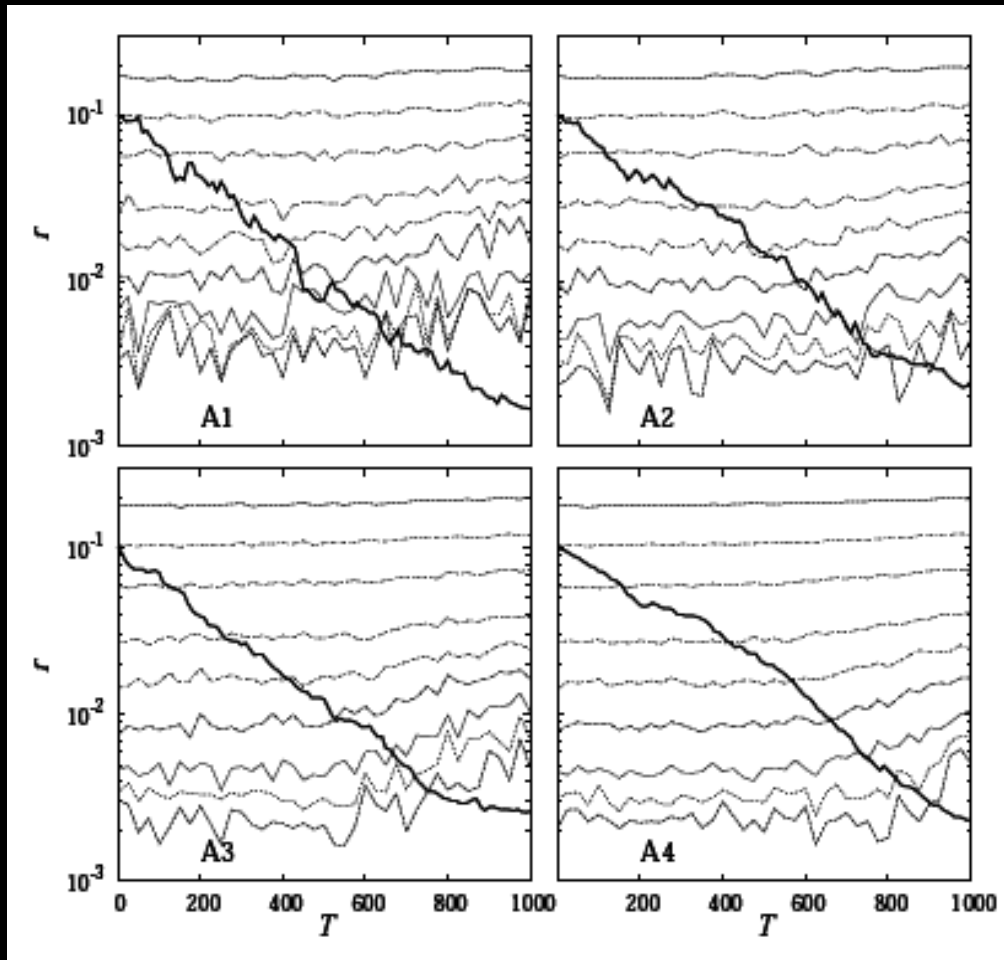
3. emission of gravitational waves (Peters 1964)

- takes over at subparsec scales
- leads the binary to coalescence

Length scales

Dynamical friction quickly drive the secondary hole down to a separation where the mass in stars enclosed in its orbit is of the order of its own mass

The cusp is modelled as a double power-law normalized to the isothermal sphere outside the radius of influence of M_1 . This defines the *lengthscale of the system*



Matsubayashi et al. 2007

$$\rho_*(r) = \frac{\sigma_*^2}{2\pi G r^2}$$

$$\rho(r) = \rho_0 \left(\frac{r}{r_0} \right)^{-\gamma}$$

$$r_0 = (3 - \gamma) \frac{GM_1}{\sigma^2}$$

The presence of a tidal disruption radius breaks the scale-freedom of the 3-body integration

For a BH of mass M_1 embedded in a cusp with slope normalized to a standard isothermal sphere outside its radius of influence we have

$$\begin{aligned} r_{\text{inf}} &= (3 - \gamma) \frac{GM_1}{\sigma_*^2} \simeq 4.6 \text{ pc } (3 - \gamma) M_7 \sigma_{100}^{-2}, \\ a_0 &= q^{1/(3-\gamma)} r_{\text{inf}}, \end{aligned}$$

The tidal disruption radius is defined as

$$\begin{aligned} r_t &= r_* \left(\frac{M_{\text{BH}}}{M_*} \right)^{1/3} \\ &\simeq 4.7 \times 10^{-6} \text{ pc } \left(\frac{r_*}{R_\odot} \right) \left(\frac{M_*}{M_\odot} \right)^{-1/3} \left(\frac{M_{\text{BH}}}{10^7 M_\odot} \right)^{1/3} \end{aligned}$$

The ratio of the two relevant length scales is

$$\begin{aligned} \frac{r_{t1}}{a_0} &= \frac{\sigma_*^2 r_* q^{-1/(3-\gamma)}}{Gm_* (3 - \gamma)} \left(\frac{M_1}{m_*} \right)^{-2/3} \\ &\simeq 1.0 \times 10^{-6} (3 - \gamma)^{-1} M_7^{-2/3} q^{-1/(3-\gamma)} \sigma_{100}^2 \left(\frac{r_*}{R_\odot} \right) \left(\frac{m_*}{M_\odot} \right)^{-1/3} \end{aligned}$$

We normalize our experiment to the case $a=a_0$, $M_1=10^7 M_\odot$, $q=1/81$, $\gamma=2$

We record the first passage of each star in 20 equally log spaced points in the range $(0.01-100) r_{t1}/a_0$

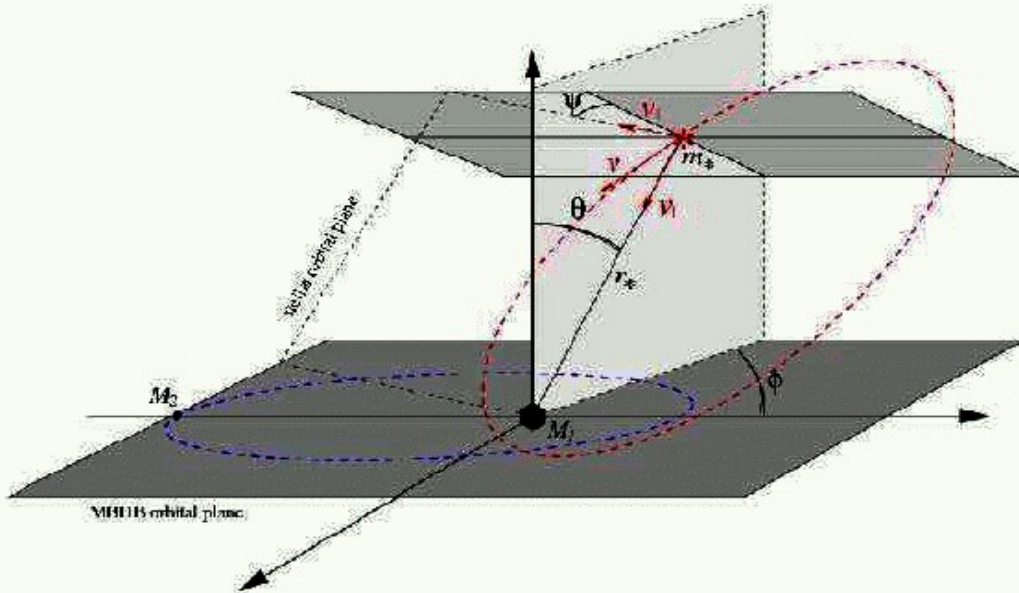
Three body scattering experiments



We integrate the nine coupled second order, differential equations

$$\ddot{\mathbf{r}}_i = -G \sum_{i \neq j} \frac{m_j (\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$

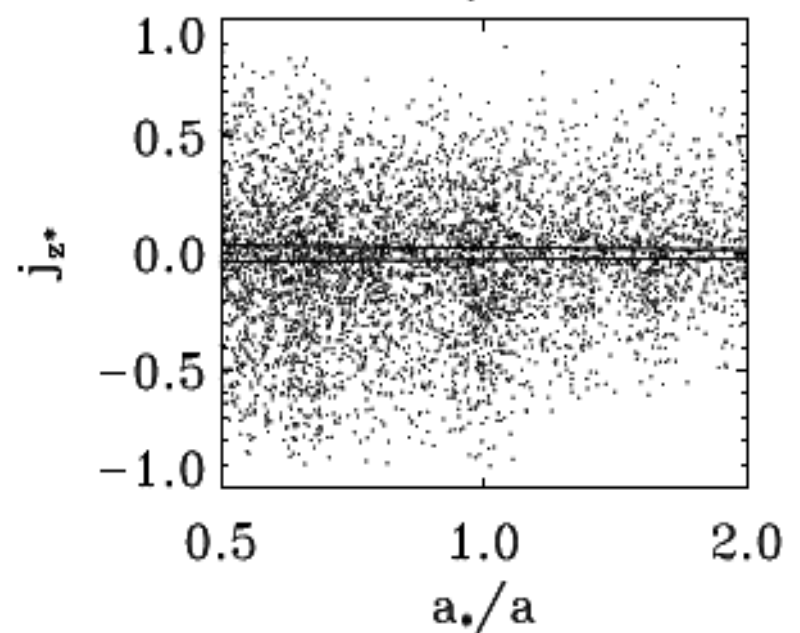
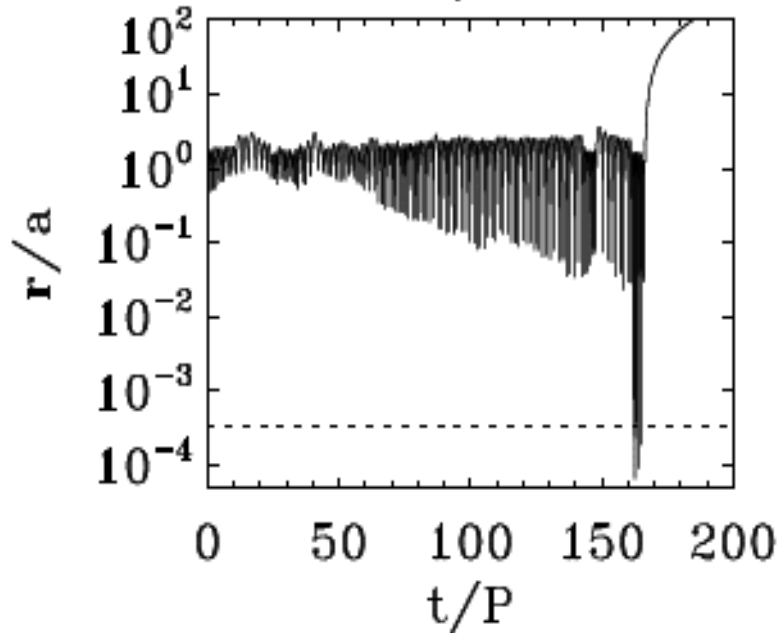
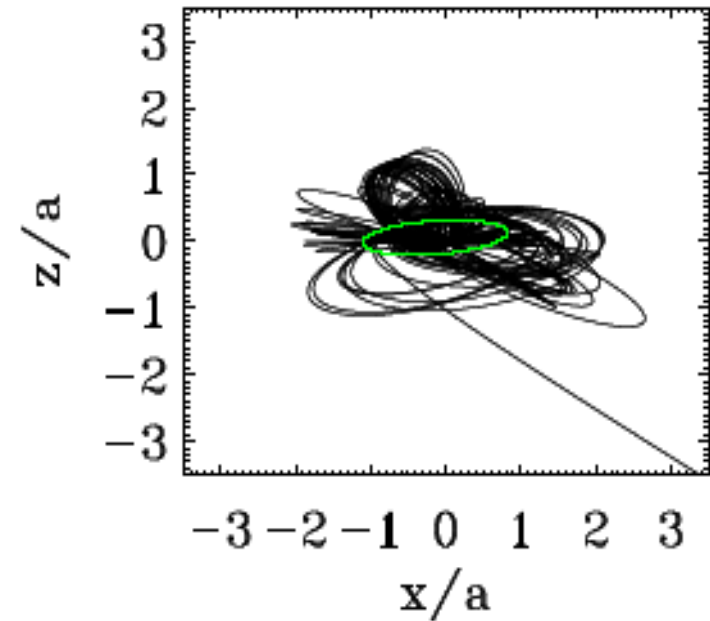
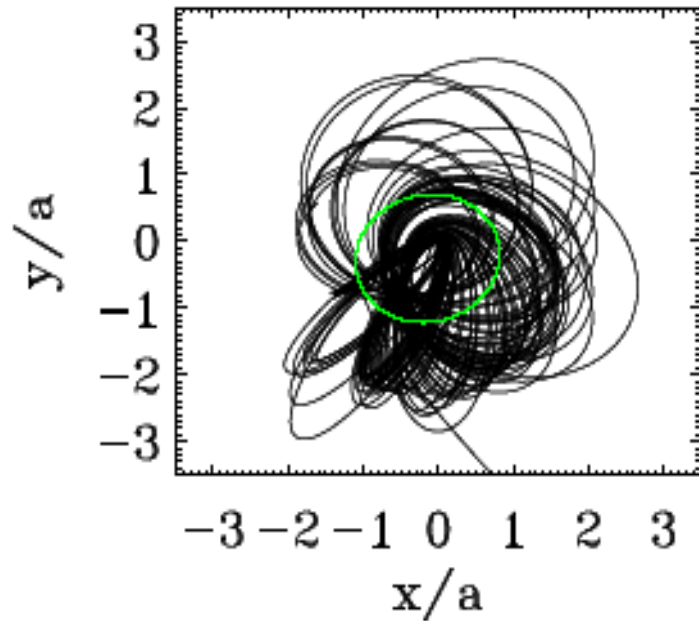
using the explicit Runge-Kutta integrator DOPRI8 (Hairer & Wanner 2002)



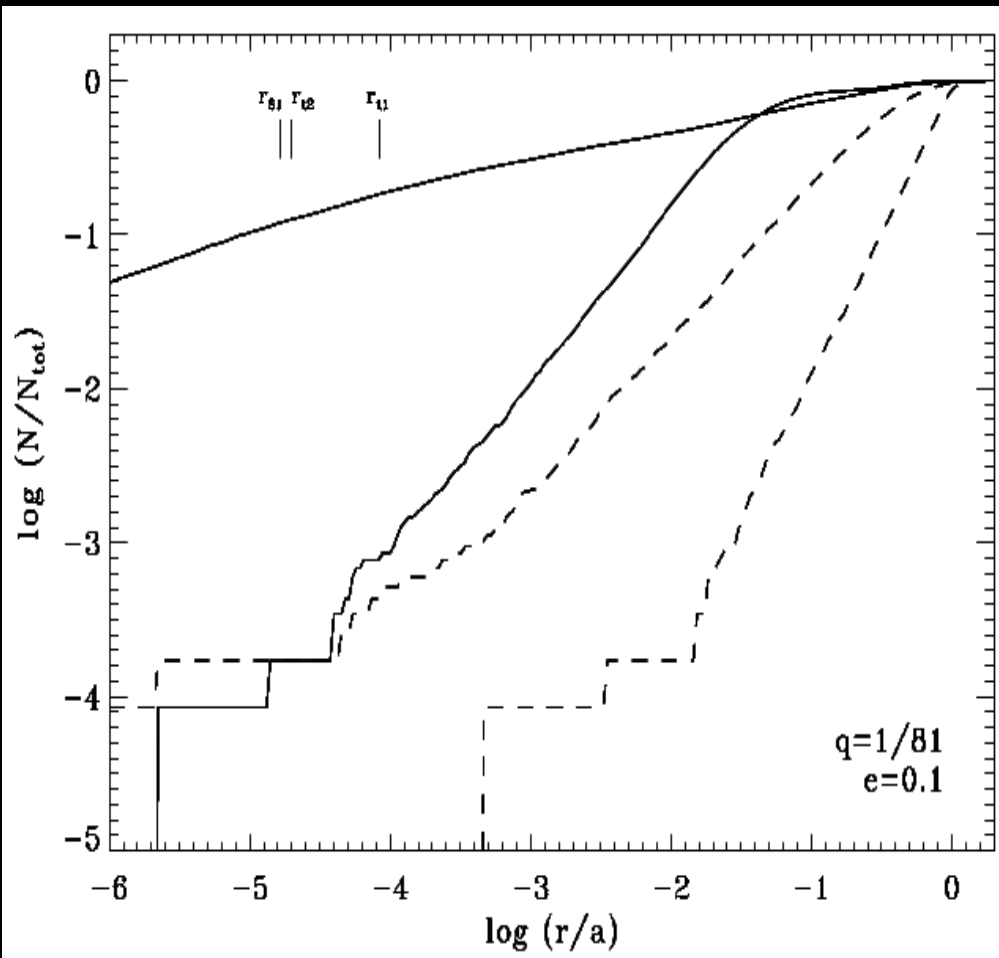
10 INITIAL CONDITIONS NEED TO BE SPECIFIED

- 1) binary mass ratio
- 2) binary eccentricity
- 3) star/BH mass ratio
- 4) initial distance of the star
- 5-6) energy and angular momentum of the stellar orbit
- 7-8-9) angles defining the star orbit
- 10) initial binary phase

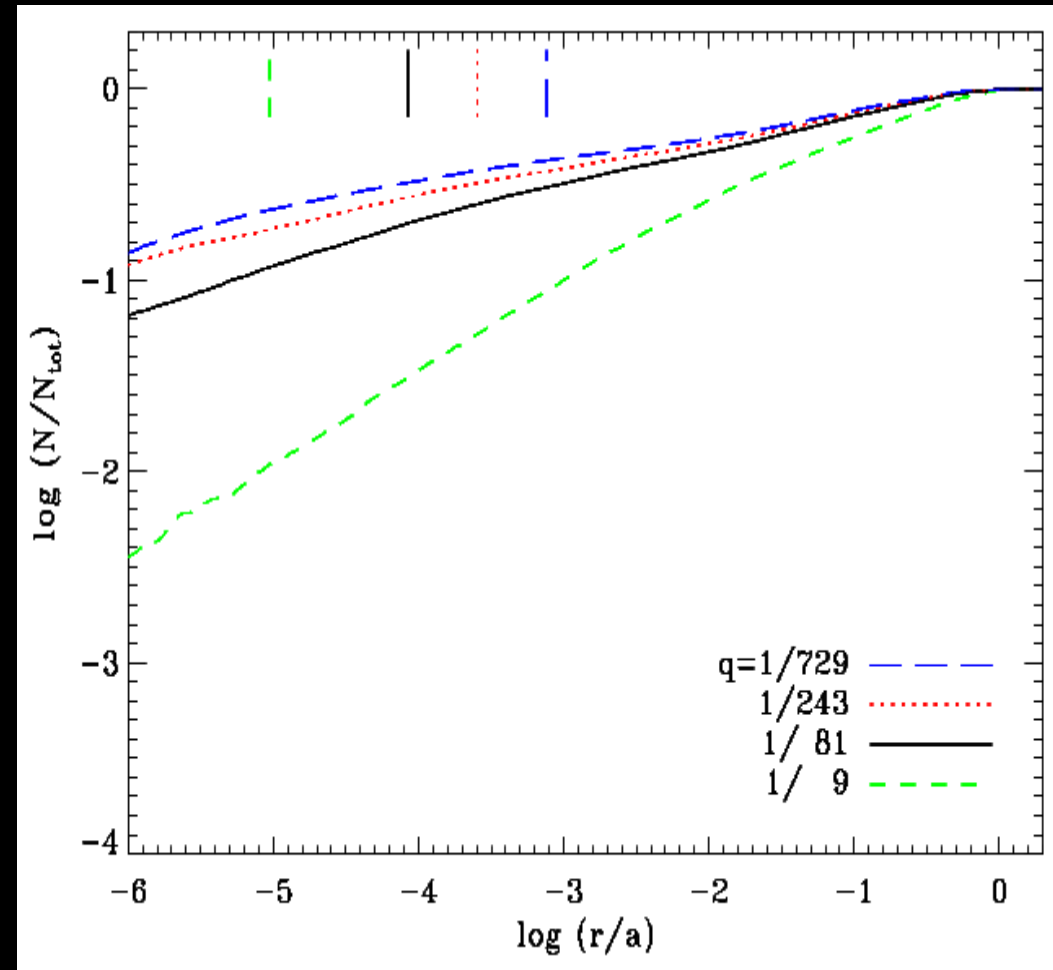
Examples of orbit integration



Tidal disruption 'cross sections'



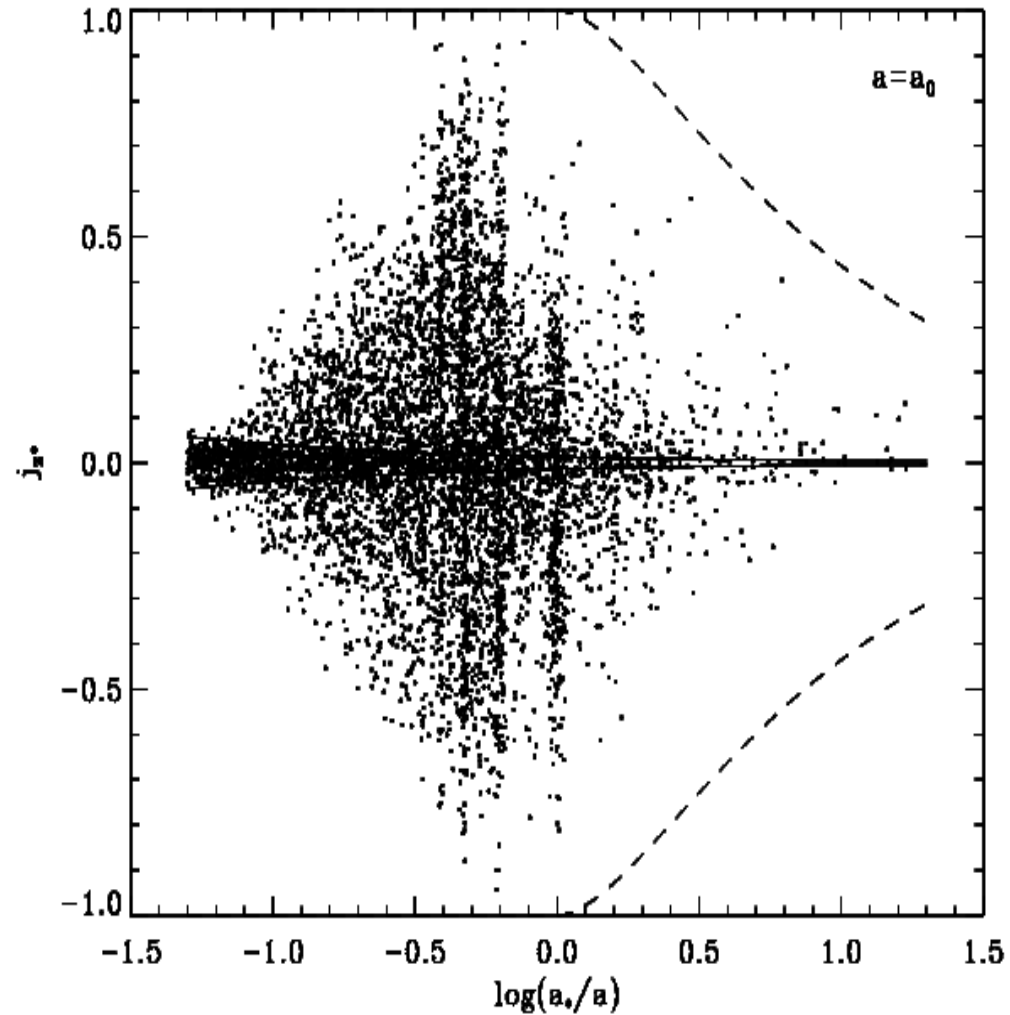
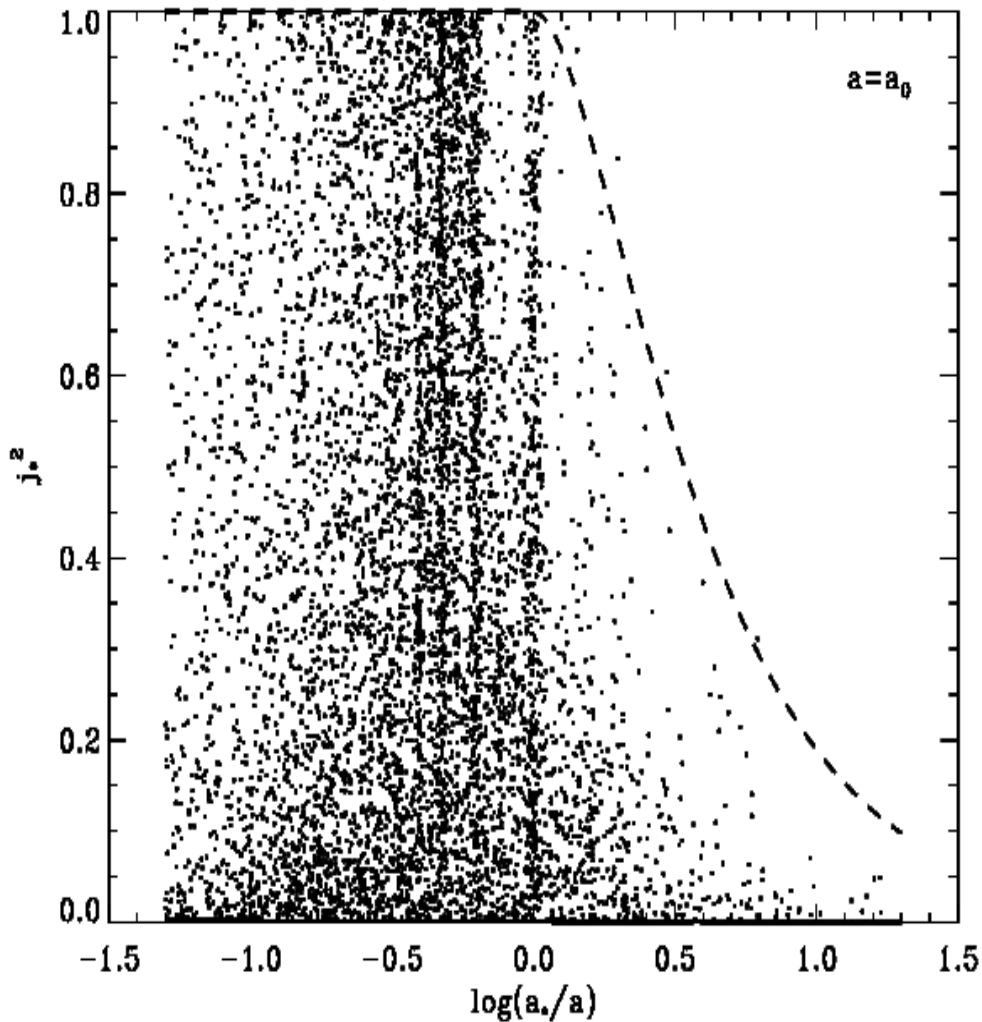
Dependence on the binary mass ratio



Binary vs single MBH cross sections:
Tidal disruption cross section increases by 3 orders of magnitude

Phase space structure of the disrupted stars

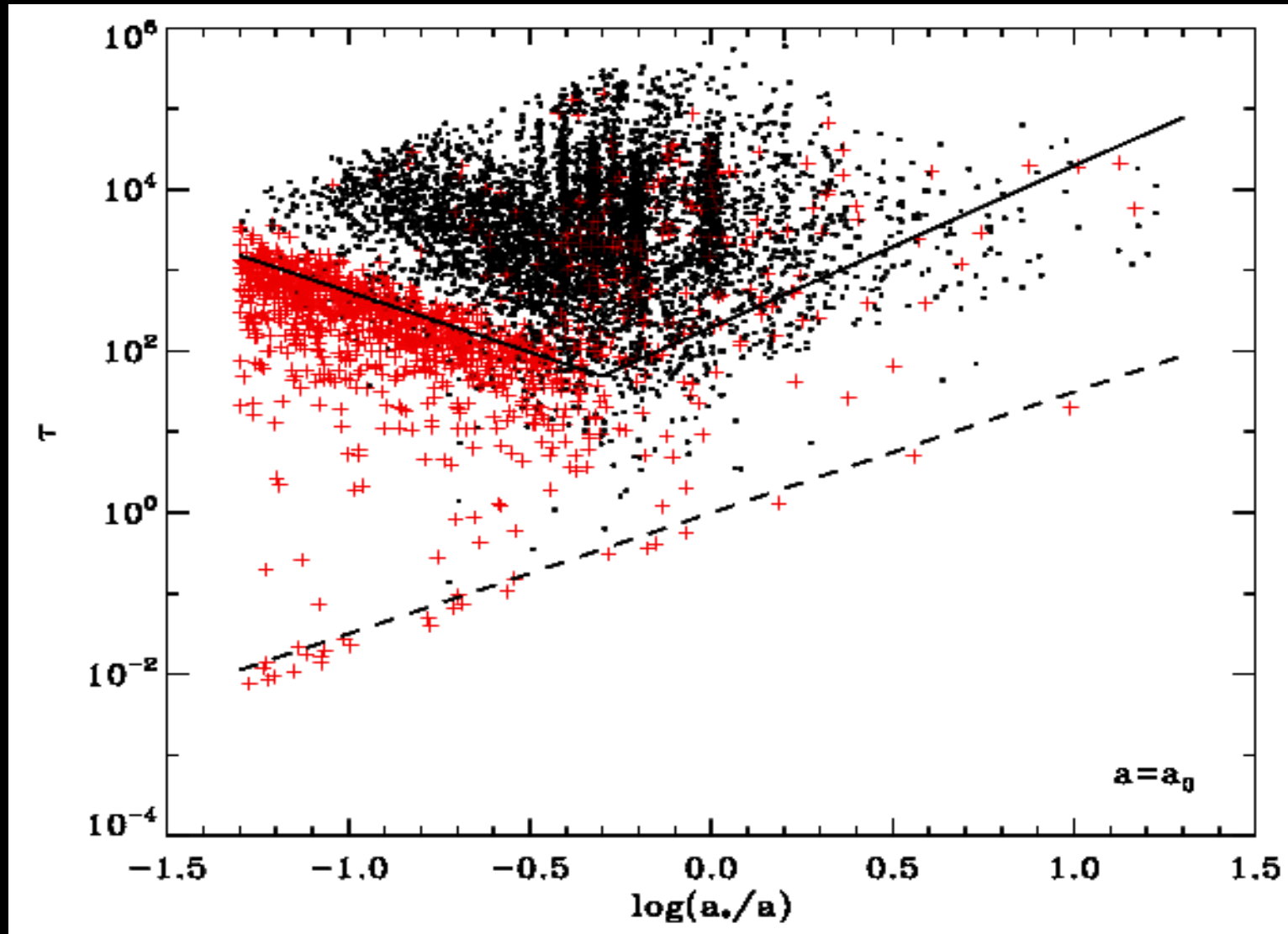
No clear structure
in the j^2 - \mathcal{E} space



Structure emerging in the j_z - \mathcal{E}
space: *Kozai resonances*
+chaotic interactions

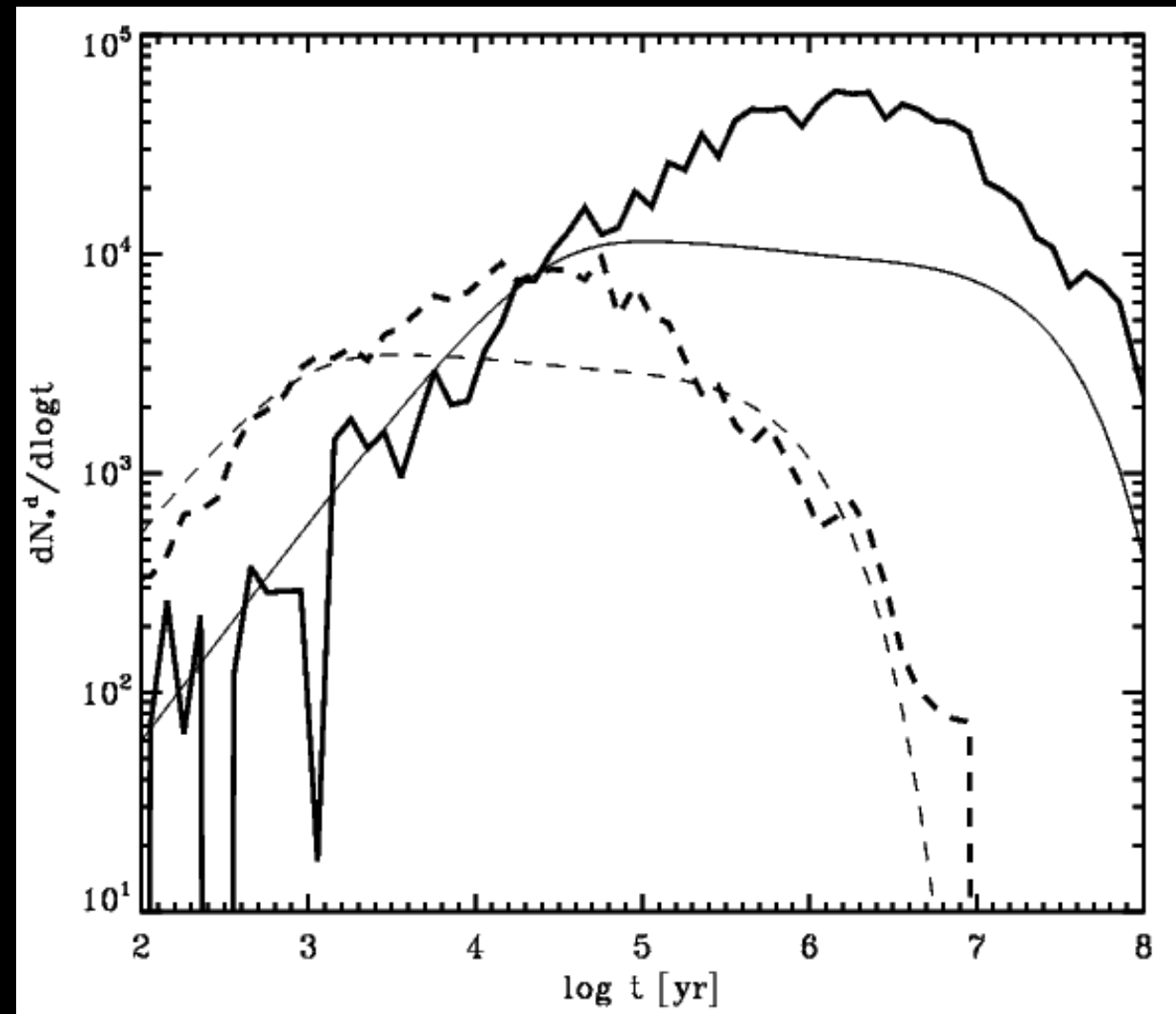
Tidal disruption timescales

$$T_K = \begin{cases} \frac{2}{3\pi q} \left(\frac{a_*}{a}\right)^{-3/2} P & (a_* \leq a/2) \\ \frac{16\sqrt{2}}{3\pi q} \left(\frac{a_*}{a}\right)^2 P & (a_* > a/2) \end{cases}$$



Static binary: tidal disruption rates

Weighting properly the tidal disruption events over an initial distribution function for the interacting stars in the cusp, the results can be translated into a *tidal disruption rate*



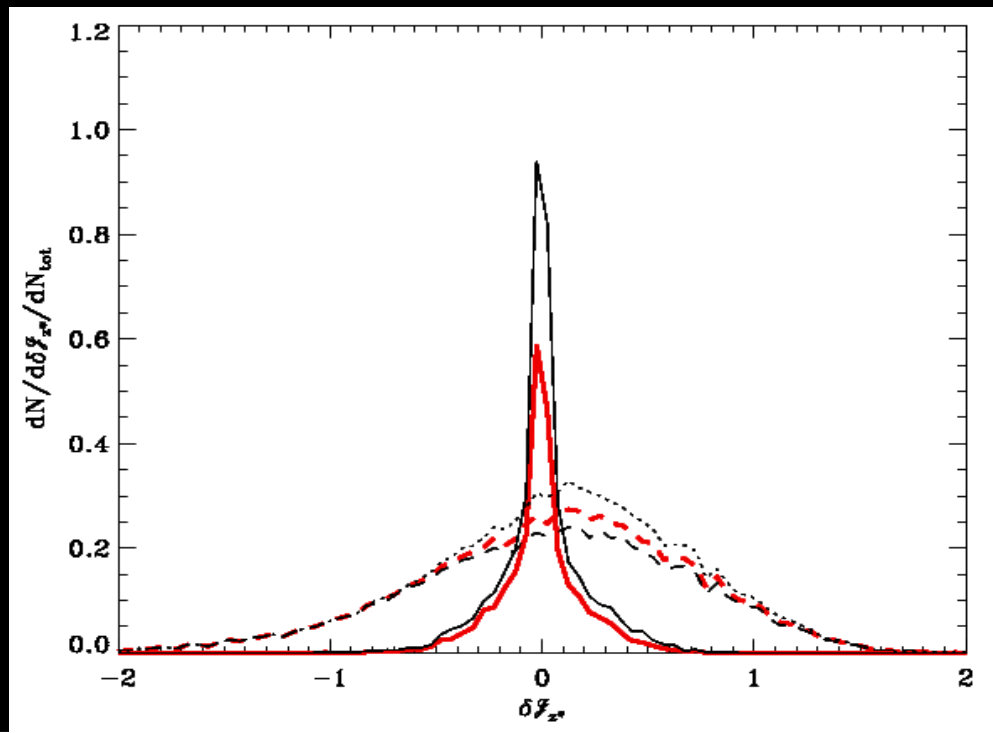
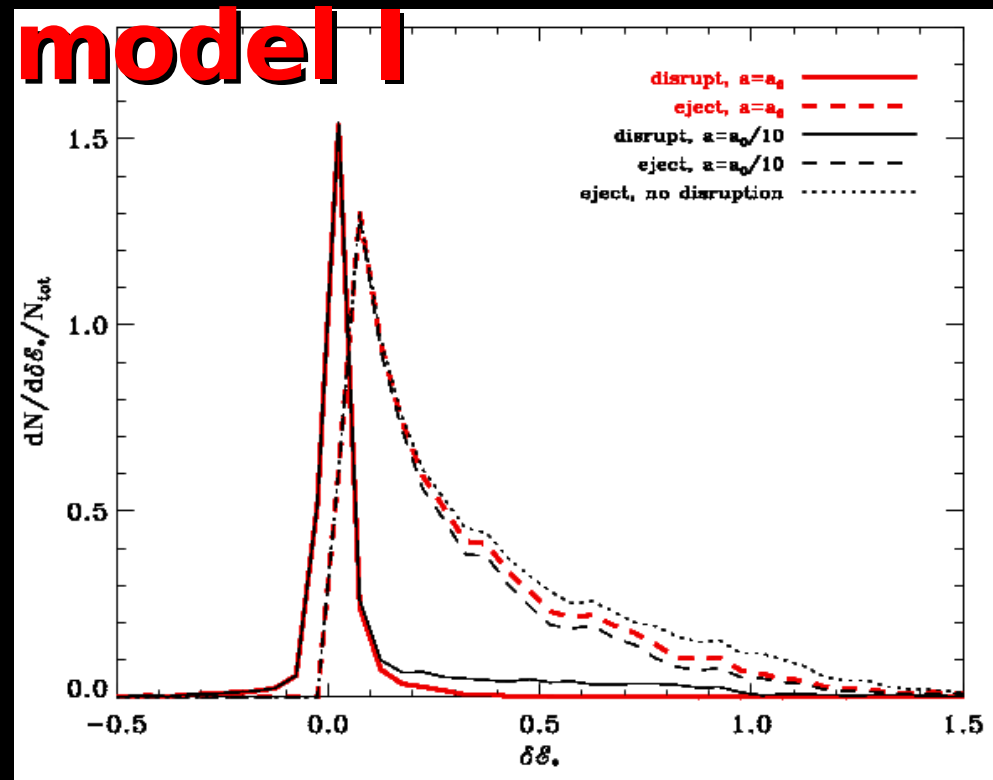
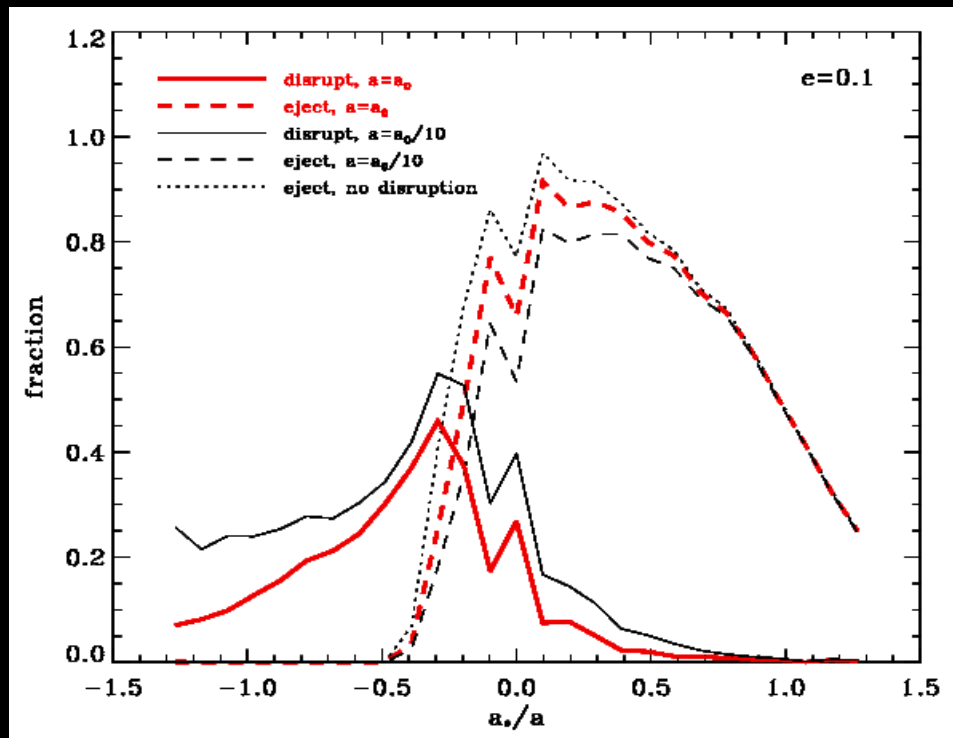
$(M_1=10^7 M_\odot, q=1/81, \gamma=2)$

TD rates for a stationary MBHB in a SIS profile.

The two set of curves are for $a=a_0$ and $a=0.1a_0$

The hybrid model I

We construct distributions of ejected and disrupted stars as a function of time on a grid of a/a_0 and e



The hybrid model II

Stars are drawn from an initial distribution function representing a cusp centred onto M_1 with a given slope. M_2 is initially at a_0 so that

$$M_*(r < a_0) = 2M_2$$

$$f_0(a_*, j_*, j_{z*}) = n_0(a_*) = \frac{2(3 - \gamma)}{a_0} \frac{M_2}{M_\odot} \left(\frac{a_*}{a_0} \right)^{2-\gamma}$$

We divide the a_* axis into logarithmic bins and we compute the initial mass in each bin

We evolve the binary according to:

$$\Delta m_i^{\text{eff}}(\Delta t) = \Delta m_i(\tau_i + \Delta t/P) - \Delta m_i(\tau_i),$$

$$\Delta \mathcal{E}_b(\Delta t) = \sum_{i=1}^{100} [\Delta \mathcal{E}_i(\tau_i + \Delta t/P) - \Delta \mathcal{E}_i(\tau_i)],$$

$$\Delta \mathcal{J}_b(\Delta t) = \sum_{i=1}^{100} [\Delta \mathcal{J}_i(\tau_i + \Delta t/P) - \Delta \mathcal{J}_i(\tau_i)].$$

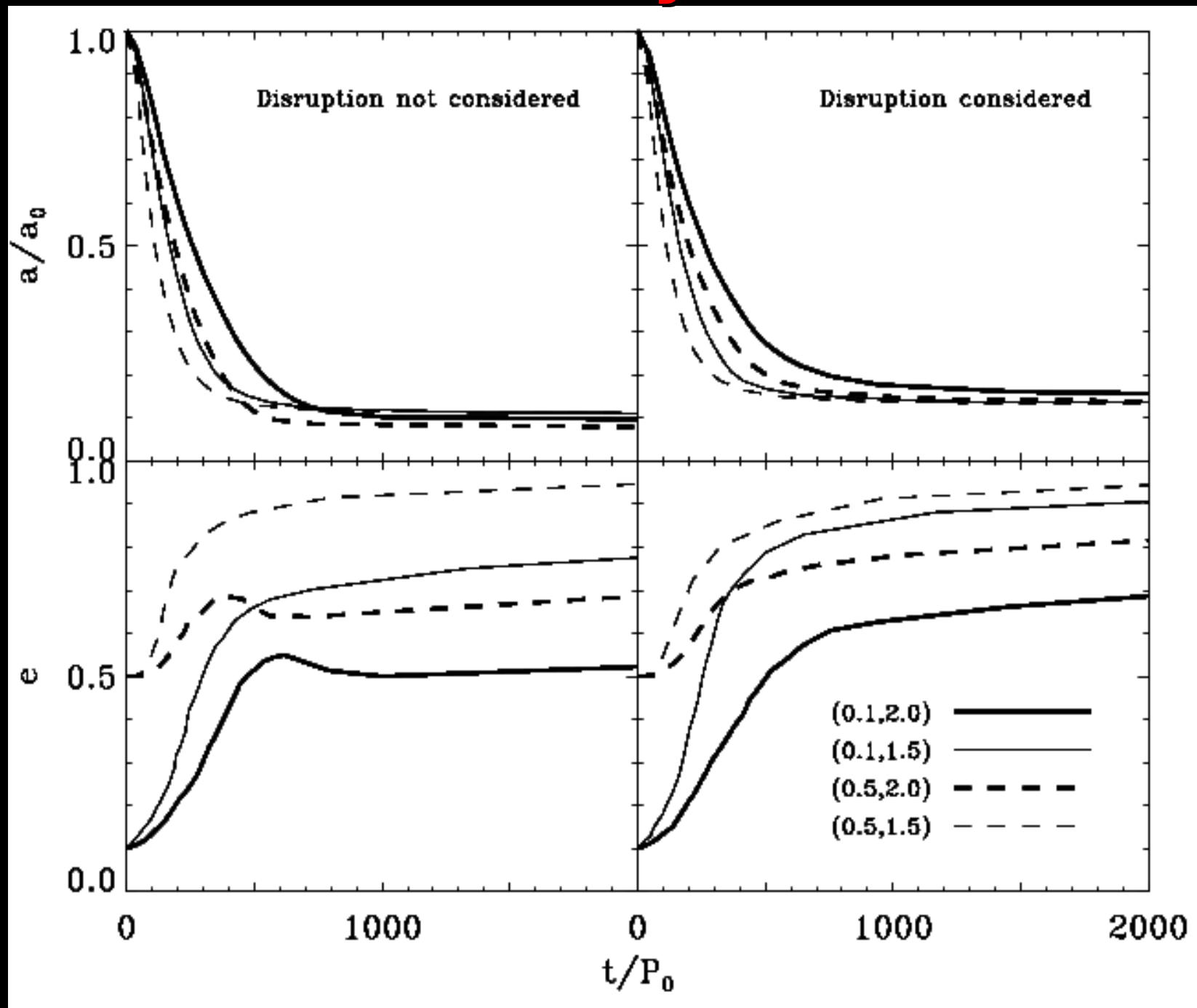
$$m'_i = m_i - \Delta m_i^{\text{eff}}(\Delta t),$$

$$a' = a - \frac{\Delta \mathcal{E}_b(\Delta t)}{\mathcal{E}_b} a,$$

$$e' = e - \frac{1 - e^2}{2e} \left[\frac{\Delta \mathcal{E}_b(\Delta t)}{\mathcal{E}_b} + \frac{2\Delta \mathcal{J}_b(\Delta t)}{\mathcal{J}_b} \right]$$

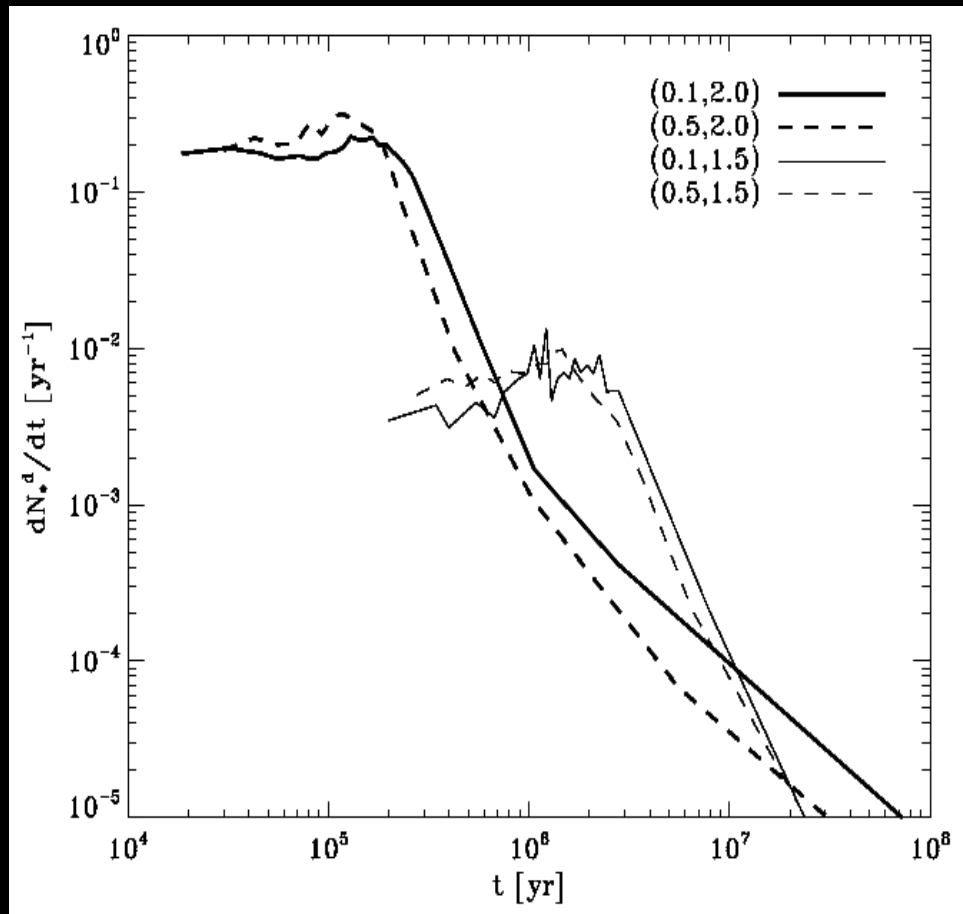
At each timestep the number of ejected and disrupted stars computed interpolating among the grids and the stellar profile is accordingly updated

Results: binary evolution



$(M_1=10^7 M_\odot, q=1/81, \gamma=2)$

Results: tidal disruption rates



**Tidal disruption rate for an
evolving MBHB**
($M_1=10^7 M_\odot$, $q=1/81$)

The *peak tidal disruption rate is*

$R \sim 0.3/\text{yr}$
scaling as

$$\dot{N}_*^d \propto (3 - \gamma)^{-2} q^{(4-2\gamma)/(3-\gamma)} \left(\frac{a}{a_0}\right)^{1-\gamma} M_1^{-1/3} \sigma_*^4$$

$$\propto (3 - \gamma)^{-2} q^{(4-2\gamma)/(3-\gamma)} \left(\frac{a}{a_0}\right)^{1-\gamma} M_1^{2/3},$$

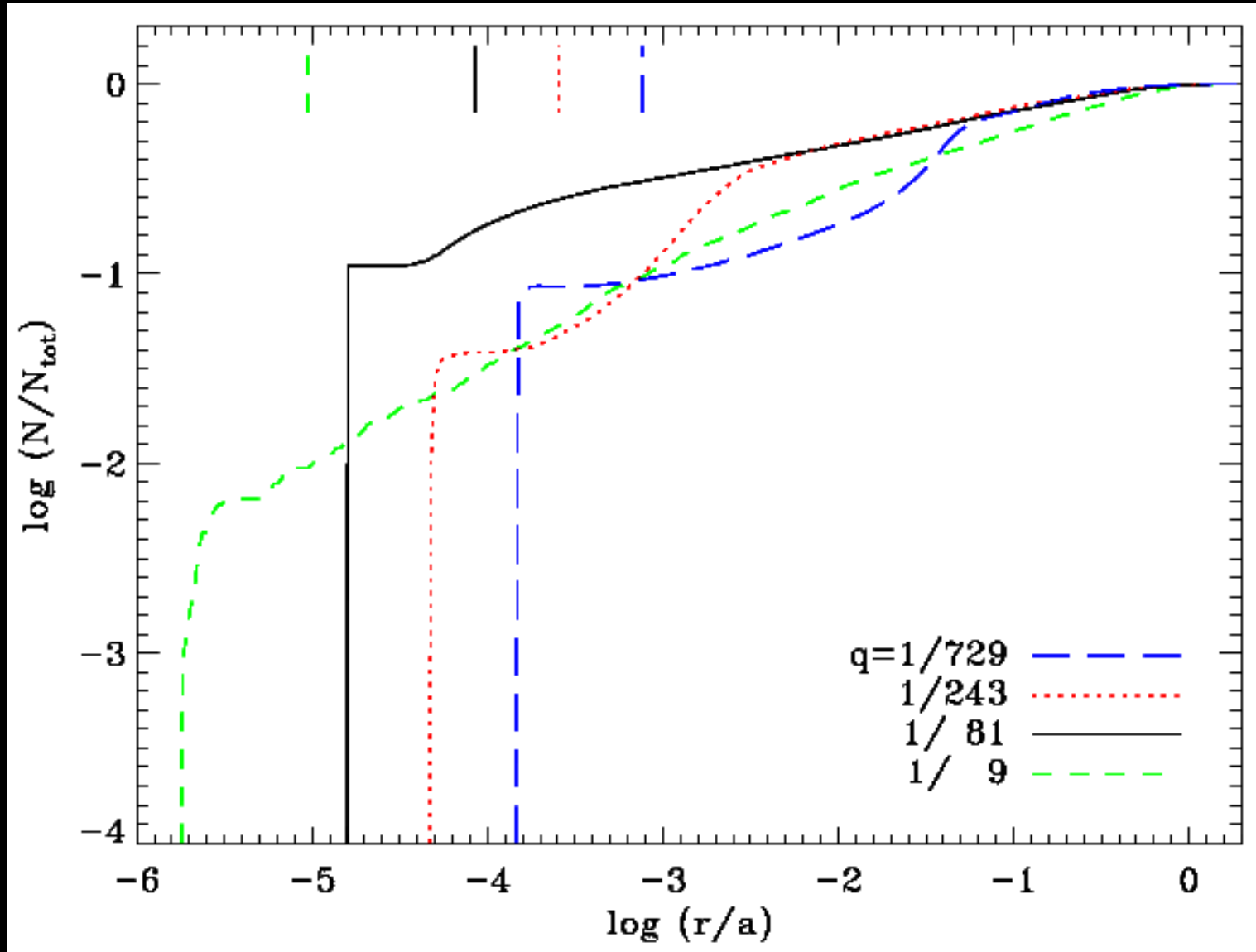
The *number of disrupted stars is*

$N \sim 5 \times 10^4$
scaling as

$$N_{\text{tot}}^d \propto t_{\text{evo}} \dot{N}_*^d \propto (3 - \gamma)^{-1/2} q^{(2-\gamma)/(6-2\gamma)} M^{2/3} \sigma$$

$$\propto (3 - \gamma)^{-1/2} q^{(2-\gamma)/(6-2\gamma)} M^{11/12}$$

The effect of GR

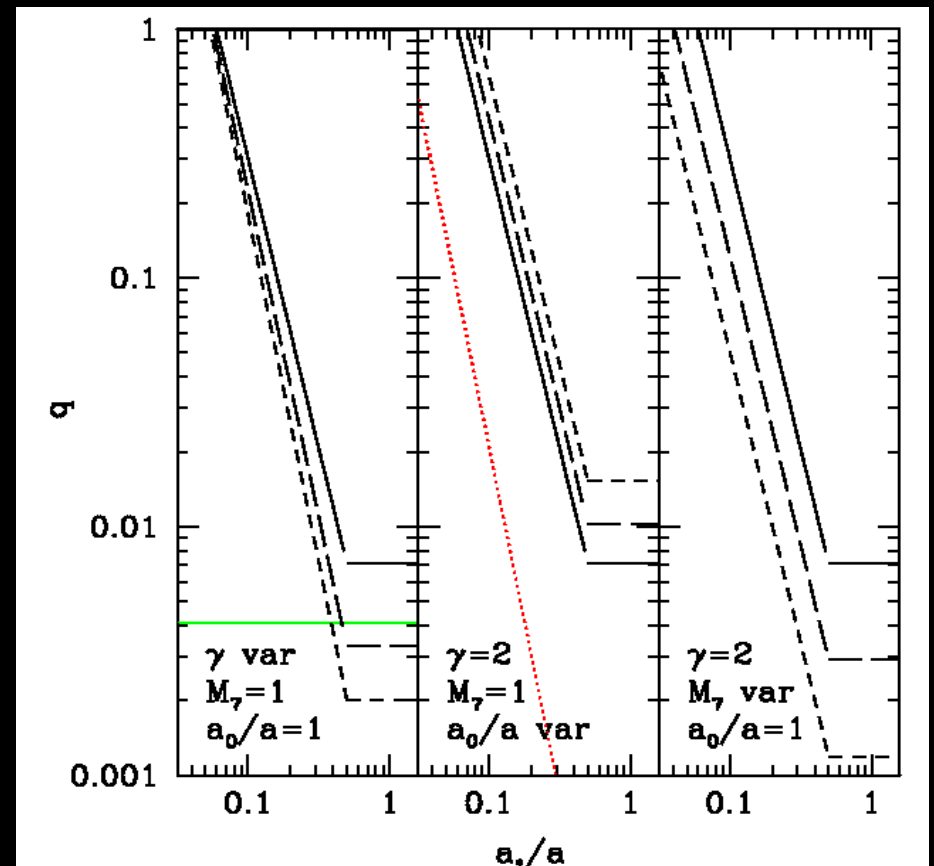
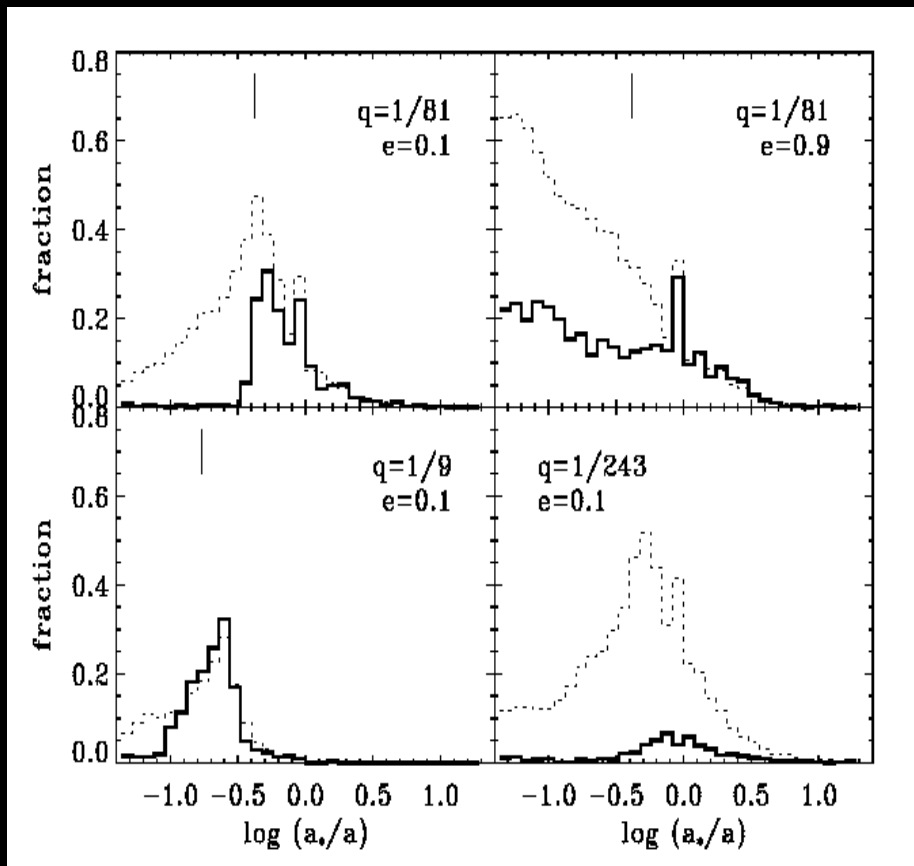


Paczynski-Witta pseudo-Newtonian potential
The disruption cross section drops significantly for $q < 0.01$

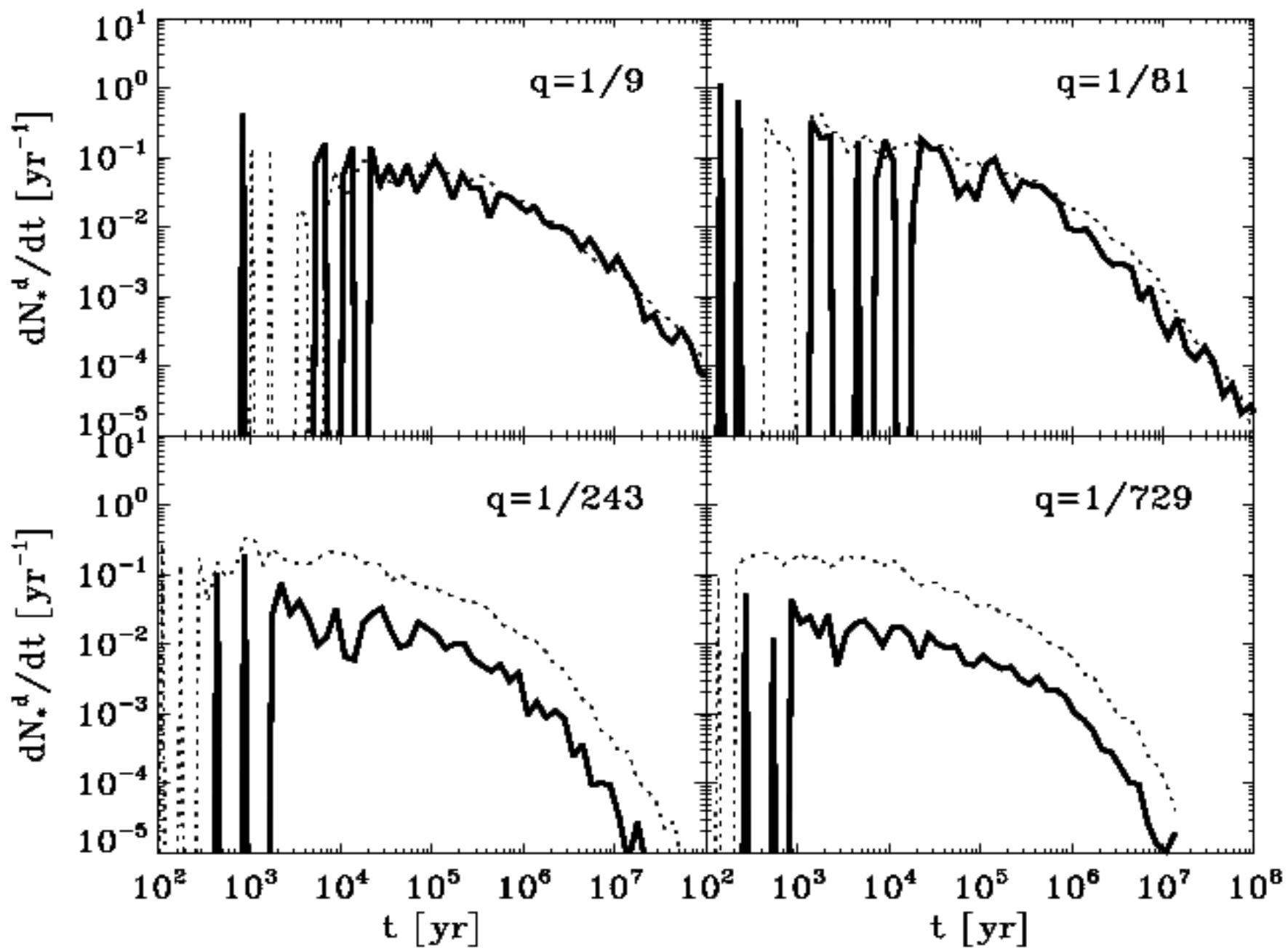
GR vs. Kozai precession

$$\dot{\omega}_K \simeq \begin{cases} \frac{15\pi q}{2\sqrt{2}P(a)} \left(\frac{r_{t1}}{a}\right)^{-1/2} \left(\frac{a_*}{a}\right)^2 & (a_* < a/2) \\ \frac{15\pi q}{32P(a)} \left(\frac{r_{t1}}{a}\right)^{-1/2} \left(\frac{a_*}{a}\right)^{-3/2} & (a_* \geq a/2) \end{cases}$$

$$\dot{\omega}_{GR} \simeq \frac{6\pi GM_1}{(1-e_*^2)c^2 a_*} P(a_*)^{-1} = \frac{3\pi}{2P(a)} \left(\frac{r_{g1}}{r_{t1}}\right) \left(\frac{a_*}{a}\right)^{-3/2}$$



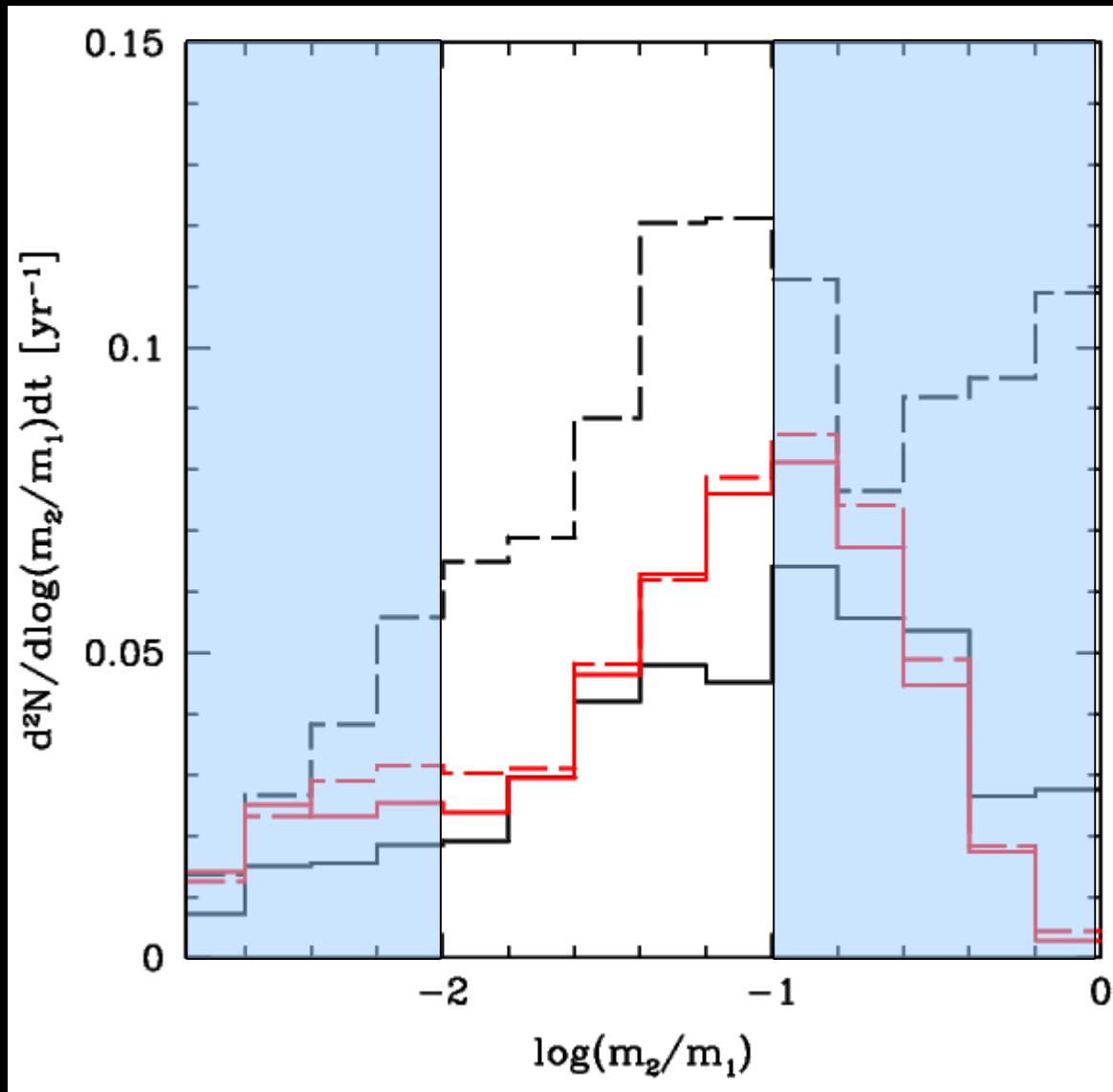
Disruption rate suppression



Number of events

$\sim 5 \times 10^4 M/M_\odot$ disrupted stars, if $0.01 < q < 0.1$,

numbers are weakly dependent on the cusp slope



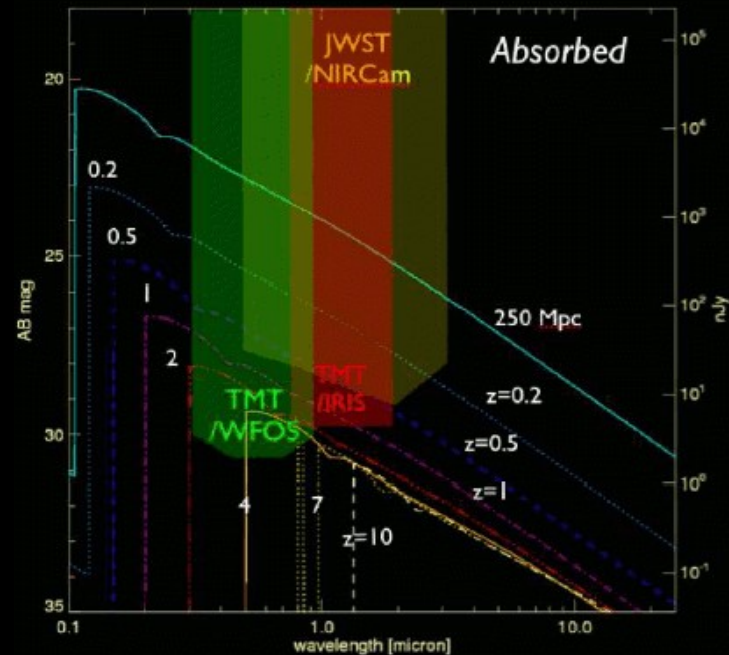
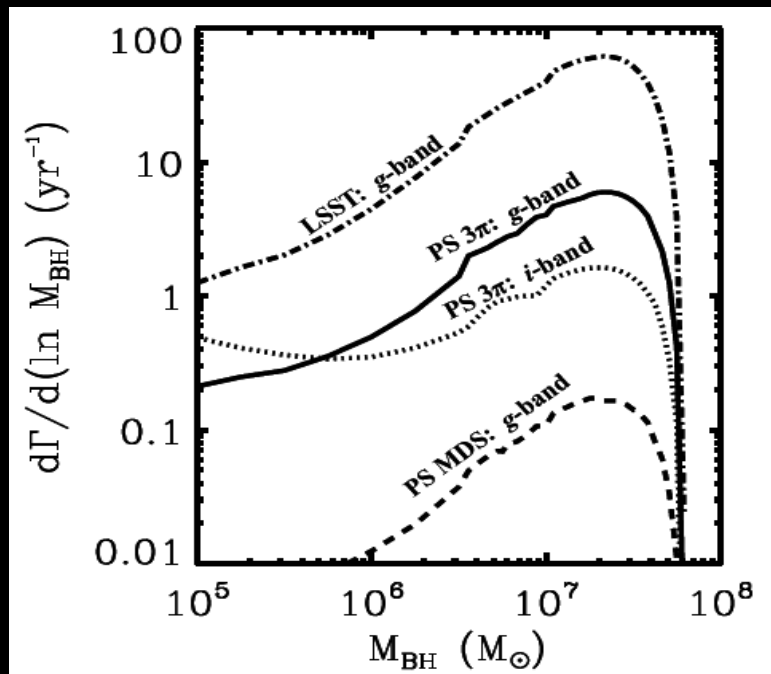
Cosmological models for MBHB formation and evolution predict a coalescence rate of $\sim 0.1/\text{yr}$ in the relevant M and q range, at $z < 1$

We thus may expect $\sim 10^3$ flaring events to be associated with MBHBs at $z < 1$

Observational probe of MBHBs?

TD flares may provide an efficient way to discover MBHBs:

- 1-TD rates for individual MBHBs implies $\sim \text{few} \times 10^5$ disruption events per galaxy per Hubble time. If all the galaxies experienced 1 minor merger in their lifetime, *as many as 10% of identified flares may be associated to MBHBs.*
- 2-In dense cusps, rates may be higher than 0.1/yr: the detection of a *recursive flare activity* in the same galaxy may provide evidence of a MBHB
- 3-If rates are $>0.1/\text{yr}$, the accretion episodes related to subsequent events may overlap, giving origin of a *short living (10^5 yr) AGN-type activity*



Summary

- > unequal MBHBs in dense stellar cusp produce a boost in the tidal disruption rate of main sequence stars
- > A tidal disruption rate as high as 0.1/yr can be sustained for $\text{few} \times 10^5$ yrs
- > GR mitigates the tidal disruption boost for $q < 0.01$
- > For $0.01 < q < 0.1$, 10^3 - 10^5 stars may be disrupted in 10^5 - 10^7 yrs depending on M and γ .
- > LSST will detect hundreds of tidal flares, as many as 10% of which may be associated to MBHBs

The “final parsec problem”

We want MBHBs to coalesce after a major merger

Dynamical friction is efficient in driving the two BHs to a separation of the order

$$a_h \simeq 0.31 \text{ pc } M_{2,6}^{1/2} \sqrt{\frac{q}{1+q}}$$

GW emission takes over at separation of the order

$$a_{GW} \approx 0.0014 \text{ pc } \left(\frac{MM_1M_2}{10^{18.3} M_\odot^3} \right)^{1/4} F(e)^{1/4} t_9^{1/4}$$

The ratio can be written as

$$\frac{a_h}{a_{GW}} \approx 2.5 \times 10^2 \left(\frac{q}{1+q} \right)^{3/4} F(e)^{-1/4} M_6^{-1/4} t_9^{-1/4}$$

A possible solution: Gravitational Slingshot

Extraction of binary binding energy via three body interaction with stars



3-body Scattering experiments

(e.g. Mikkola & Valtonen 1992, Quinlan 1996)



N-body simulation

(e.g. Milosavljevic & Merritt 2001)

- > More feasible
- > need a large amount of data for significant statistics (eccentricity problem)
- > warning: connection with real galaxies!
 - > initial conditions
 - > loss cone depletion
 - > contribution of returning stars
 - > presence of bound stellar cusps

Hardening in a fixed background

Quinlan 1996

$$H = \frac{\sigma}{G\rho_0} \frac{d}{dt} \left(\frac{1}{a} \right)$$

HARDENING RATE

$$K = \frac{de}{d \ln(1/a)}$$

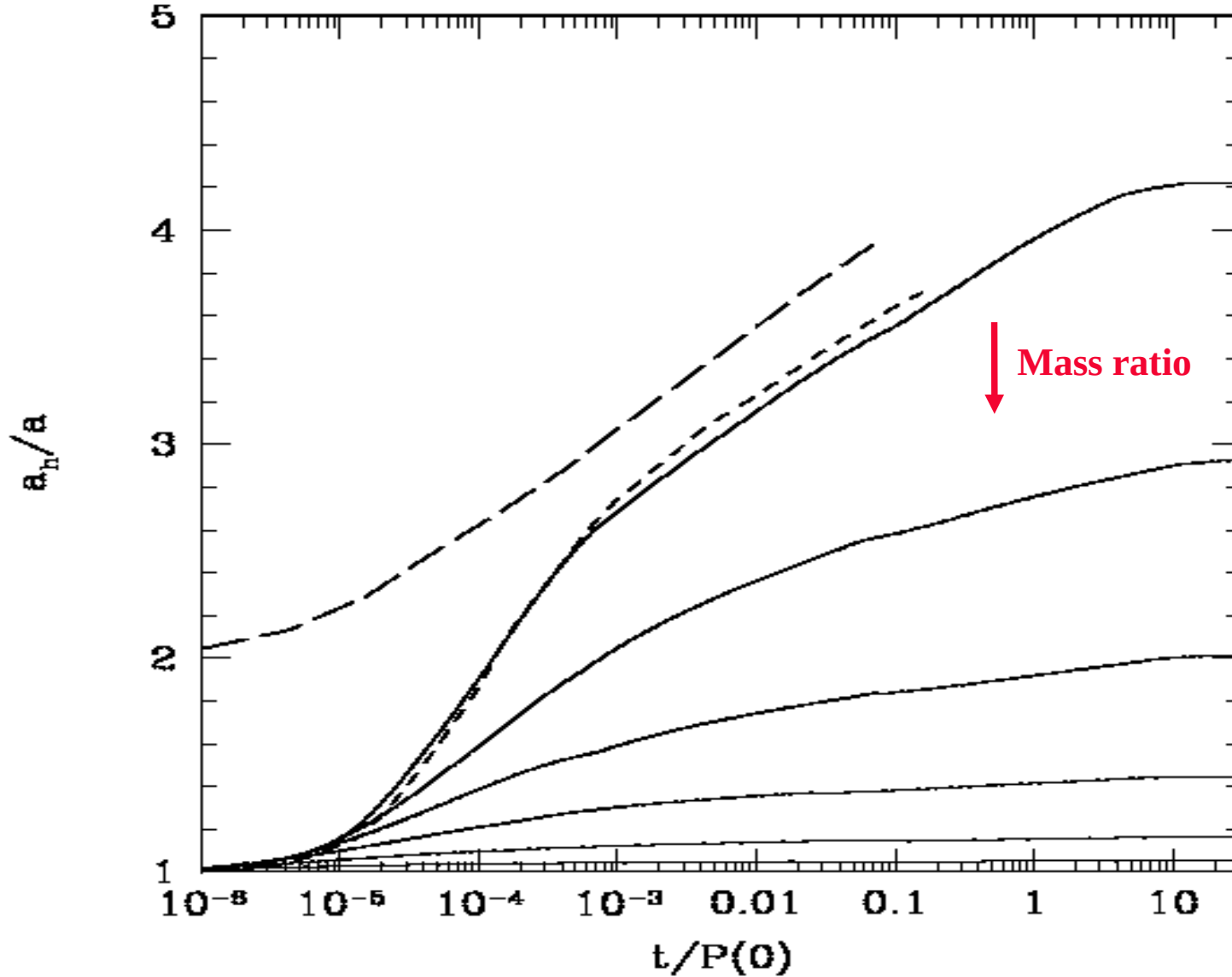
ECCENTRICITY GROWTH RATE

$$J = \frac{1}{M} \frac{dM_{ej}}{d \ln(1/a)}$$

MASS EJECTION RATE

$$v_{esc} \equiv \sqrt{-2\phi} - 2\sigma \sqrt{[\ln(M_E/M) + 1]} - 5.5\sigma$$

a-Orbital decay



- 1) orbital decay is at most a factor of ~ 5
- 2) loss cone depletion is fast

The MBHB fate depends on the supply of stars

- > ***Loss cone amplification***

 - > ***Axisymmetric and triaxial potentials***

 - (e.g. Yu 2002, Merritt & Poon 2004, Berzicik et al. 2006)

 - > ***MBHB random walk***

 - (e.g. Quinlan & Hernquist 1997, Chatterjee et al. 2003)

- > ***Relaxation processes***

 - > ***Standard two body relaxation*** (Milosavljevic & Merritt 2001)

 - > ***Massive perturbers driven relaxation*** (Perets & Alexander 2007)

 - > ***Resonant relaxation*** (Hopman & Alexander 2006)

Additional sources of energy!

- > ***Extraction of potential energy from a Bound Stellar Cusp***

 - (See next slide...)

- > ***Torques exerted on the MBHB
by a gaseous disk***

 - (Armitage & Natarajan 2002, Escala et al. 2005, Dotti et al. 2006)

Why consider bound environments

For equal MBHBs, $a_h \sim a_i$ \longrightarrow The mean stellar binding energy to the binary is negligible

But: $a_h \propto M_2$, $a_i \propto M_1$

For unequal MBHBs, $a_h \ll a_i$ \longrightarrow The mean stellar binding energy to the binary cannot be neglected

binding energy contribution *important if $q < 0.1$*
and the stellar distribution is cuspy



- > Cosmological unequal MBHBs
- > non-cosmological IMBH-MBH inspirals
(e.g. from SC inspirals)

The hybrid model

We consider a MBHB with mass ratio q with initial eccentricity e_i in a power law stellar cusp $\propto r^\gamma$

The integration start at a_i , so that $M_*(a < a_i) = 2M_2$ (Matsubayashi et al. 2005)

We solve the evolution

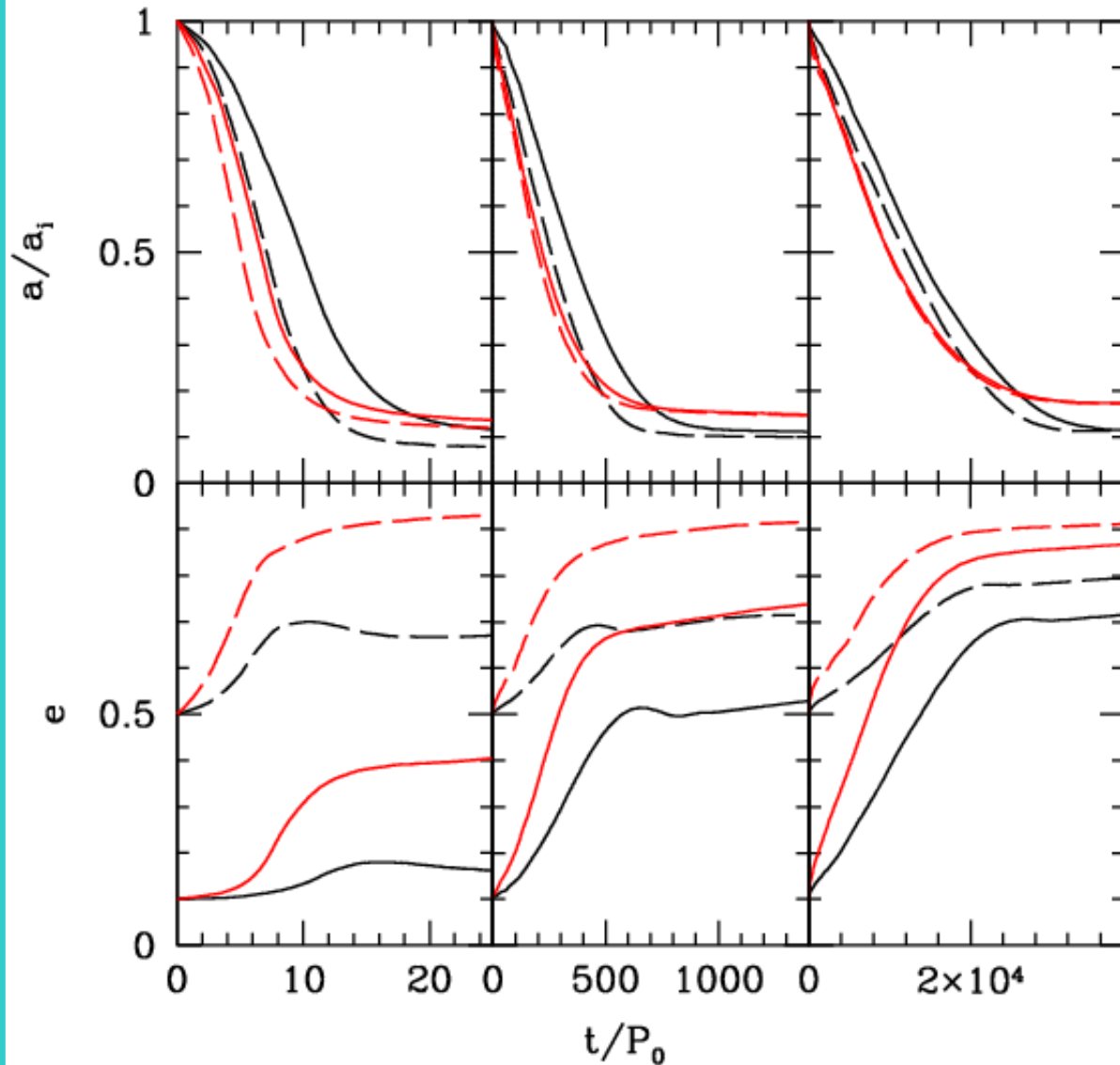
$$\frac{da}{dt} = -\frac{2a^2}{GM_1M_2} \int_0^\infty \Delta\mathcal{E} \frac{d^2 N_{ej}}{da_* dt} da_*$$

$$\frac{de}{dt} = \int_0^\infty \Delta e \frac{d^2 N_{ej}}{da_* dt} da_*$$

Δe , $\Delta\mathcal{E}$ and $d^2 N_{ej}/da_* dt$ are provided by 3-body experiments

Results I: the MBHB side

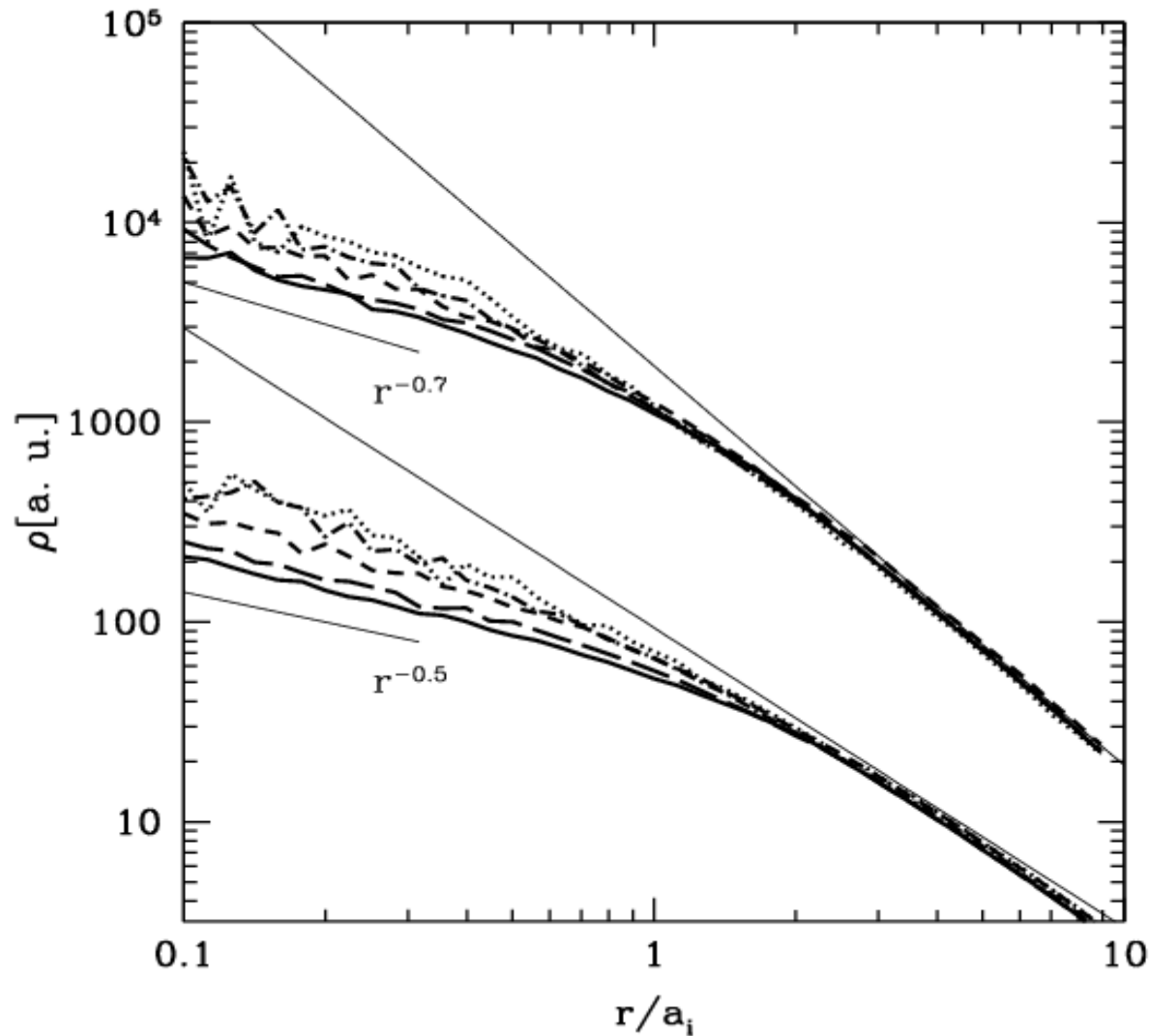
(All the results are shown in units of a_i , P_0 , $V_c(a_i)$)



		$\gamma = 1.5$		$\gamma = 1.75$		$\gamma = 2$	
q	e_i	x	e_f	x	e_f	x	e_f
1/9	0.1	9.63	0.608	9.89	0.350	11.38	0.179
	0.5	9.99	0.972	11.55	0.907	15.77	0.753
	0.9	10.06	0.998	11.80	0.992	17.73	0.969
1/27	0.1	8.06	0.691	8.35	0.532	10.19	0.408
	0.5	8.26	0.959	9.35	0.862	12.35	0.710
	0.9	8.27	0.996	9.64	0.988	14.03	0.958
1/81	0.1	6.99	0.755	7.75	0.650	9.39	0.542
	0.5	6.90	0.922	7.81	0.828	10.14	0.717
	0.9	6.89	0.996	7.81	0.974	11.00	0.937
1/243	0.1	6.49	0.906	7.28	0.805	9.30	0.688
	0.5	6.39	0.971	7.25	0.914	9.92	0.818
	0.9	6.38	0.962	7.19	0.986	10.09	0.955
1/729	0.1	6.12	0.881	6.91	0.814	8.94	0.724
	0.5	5.95	0.919	6.91	0.869	9.09	0.797
	0.9	5.94	0.977	6.92	0.953	9.41	0.900

**Shrinking factors $6 < x < 18$
MBHB eccentricity grows**

Results II: the stellar side a- density-profile evolution



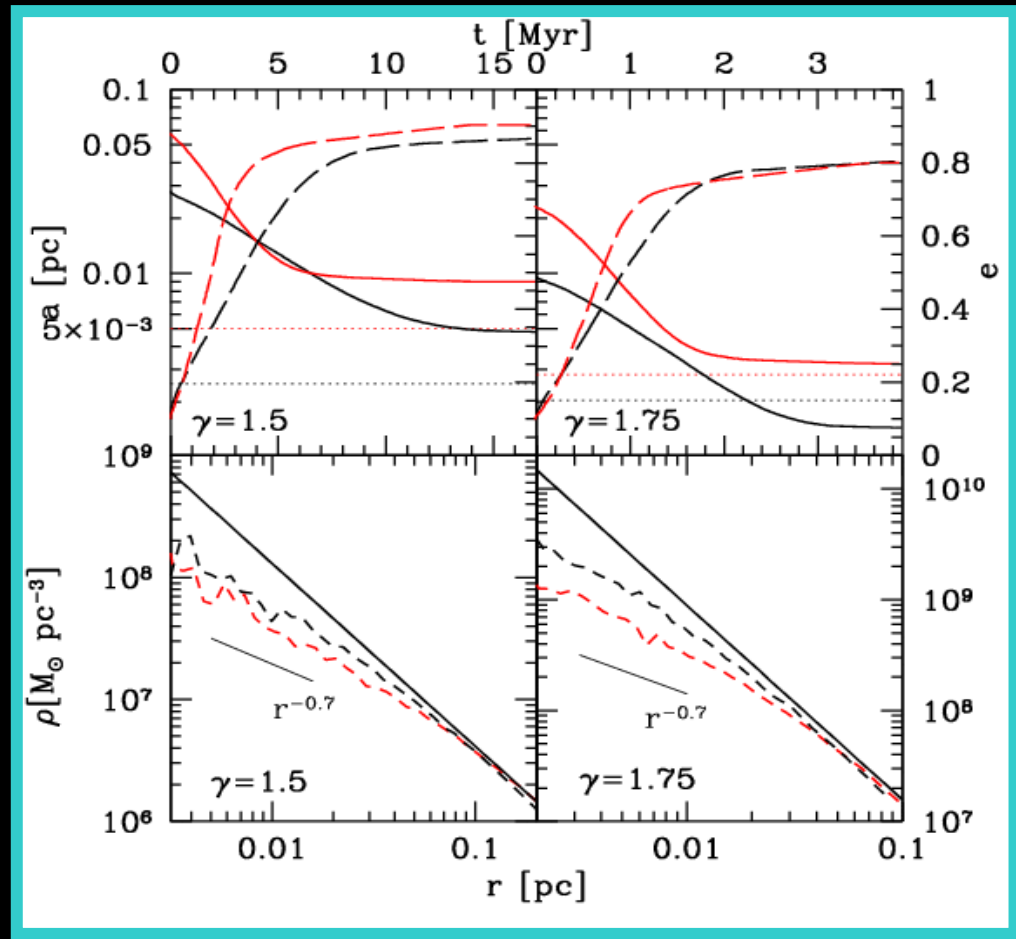
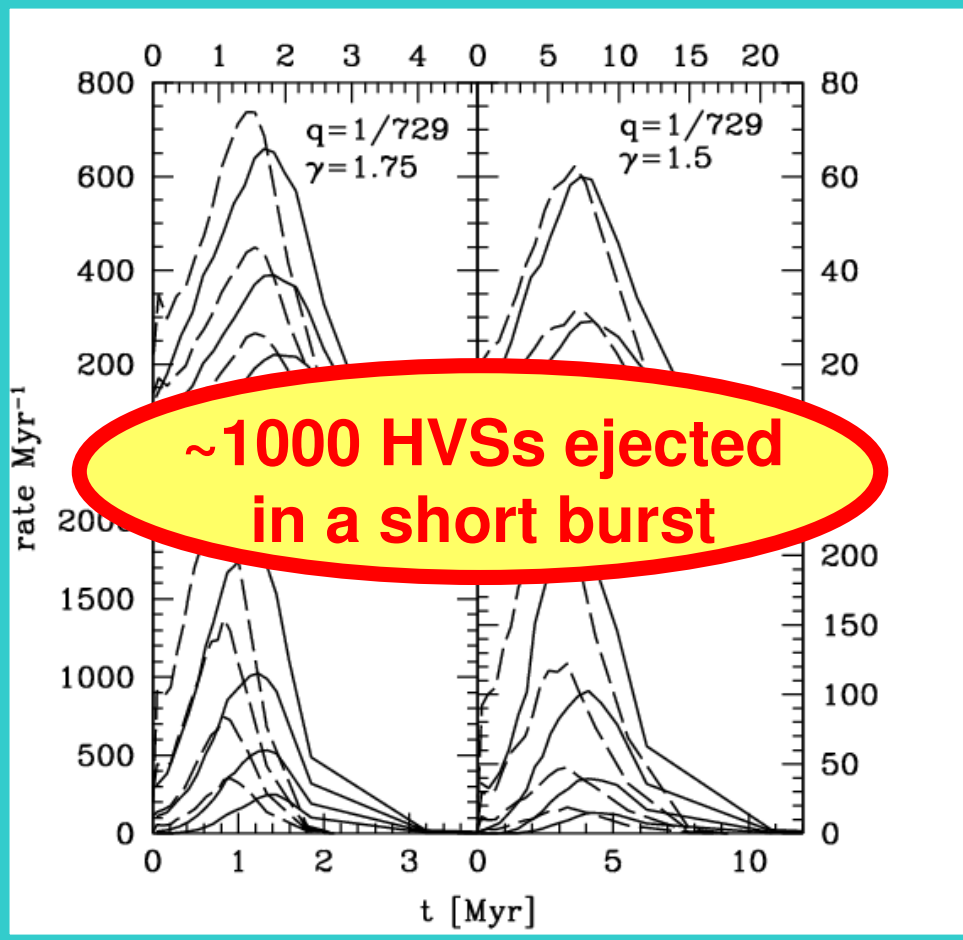
The density profile flattens significantly

The unbound mass is $2-4M_2$ almost independently on e and γ

THE CASE STUDY OF THE MILKY WAY

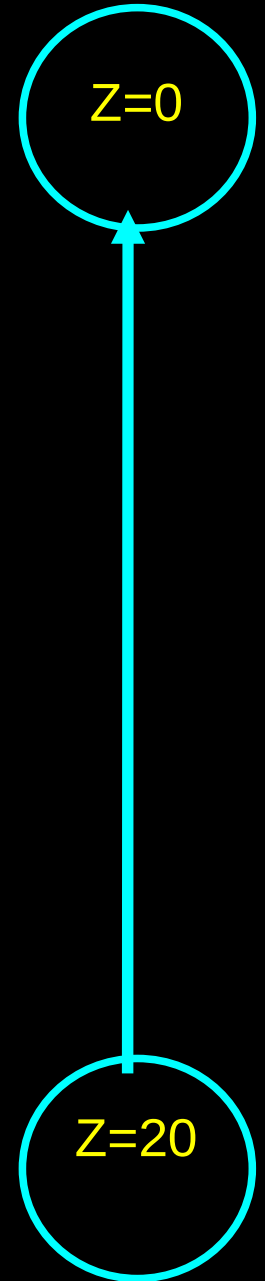
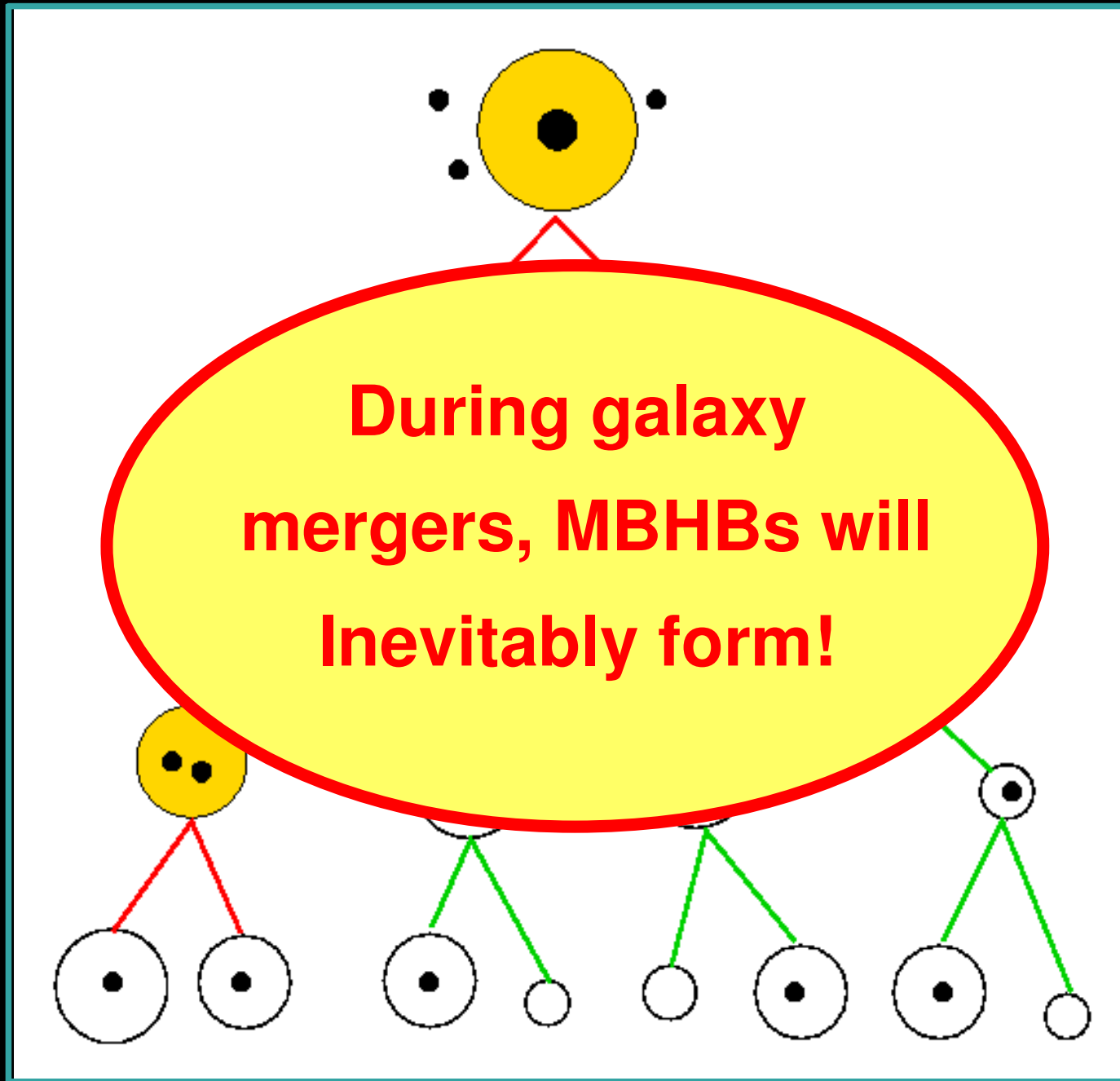
γ^a	r_0^b [pc]	ρ_0^c [$M_\odot \text{pc}^{-3}$]	q^d	a_i^e [pc]	$P_0(a_i)^f$ [yr]	$V_c(a_i)^g$ [km/s]
1.5	2.25	7.1×10^4	1/243	5.8×10^{-2}	1344	510
			1/729	2.8×10^{-2}	448	735
1.75	1.88	10^5	1/243	2.3×10^{-2}	340	806
			1/729	9.6×10^{-3}	91	1250

Large eccentricity growth during the MBHB shrinking

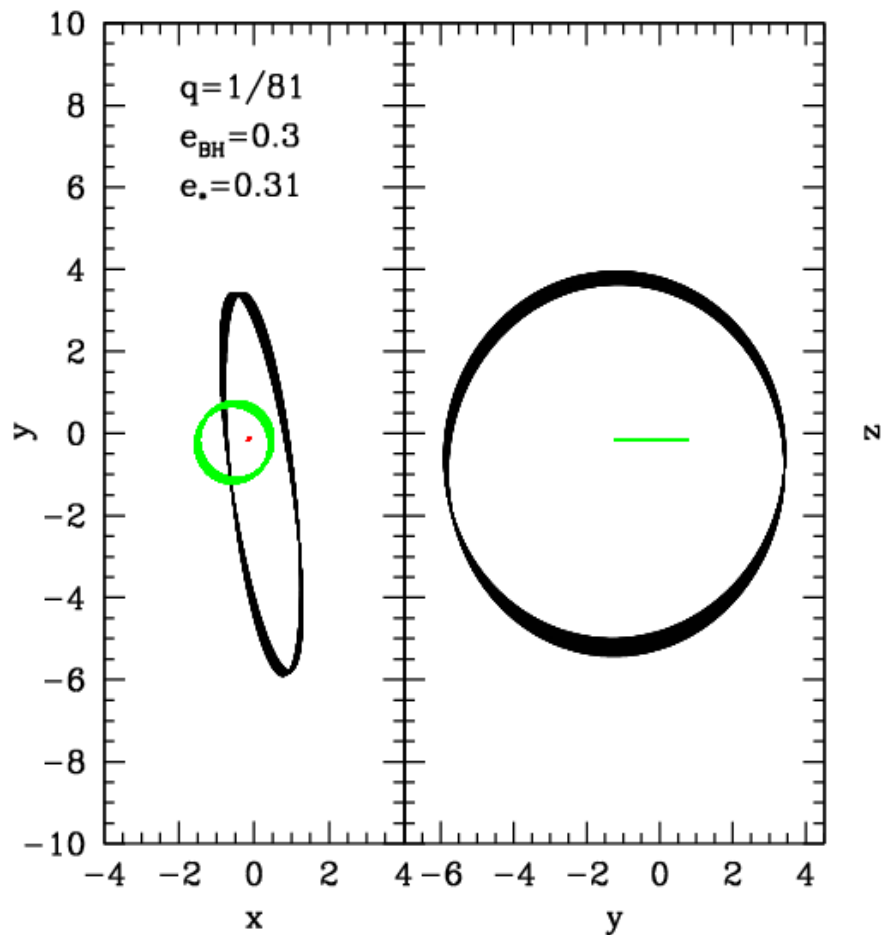


Significant flattening of the inner 10^{-2} pc

...HIERARCHICAL MBH FORMATION



Examples of orbit integrations



Weak interaction: the system behaves as a stable hierarchical triplet

Strong interaction resulting in a stellar ejection

