Numerical studies of resonant relaxation in galactic nuclei

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Outline

- What are resonant relaxation (RR) and correlation curves?
- What are the two kinds of RR?
- Measured RR coefficients
- Long term evolution of correlation curves
- Implications of RR for EMRI rates
- Preliminary relativistic results

Non-coherent relaxation vs. resonant relaxation

$$\tau = (t_2 - t_1)/P$$

$$\langle (|E_2 - E_1|/E_1)^2 \rangle^{1/2} = \Delta E \propto \sqrt{\tau}$$

$$\langle (|J_2 - J_1|/J_{c,1})^2 \rangle^{1/2} = \Delta J_s \propto \sqrt{\tau}$$ scalar

$$\langle (|\mathbf{J_2} - \mathbf{J_1}|/J_{c,1})^2 \rangle^{1/2} = \Delta J_v \propto \sqrt{\tau}$$ vector

random walk

$$\tau < \tau_\omega$$

coherent phase

approximate symmetry:

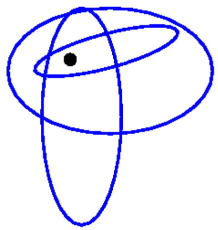
Keplerian, ellipses preserved $\Delta J_s \propto \tau$

plane of ellipses preserved $\Delta J_v \propto \tau$

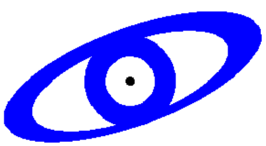
faster than random walk

Effect of scalar and vector RR

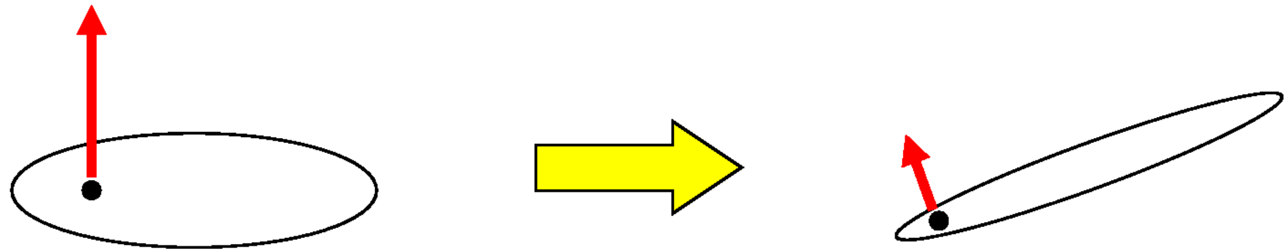
Perturbing stars	Effect on perturbed star



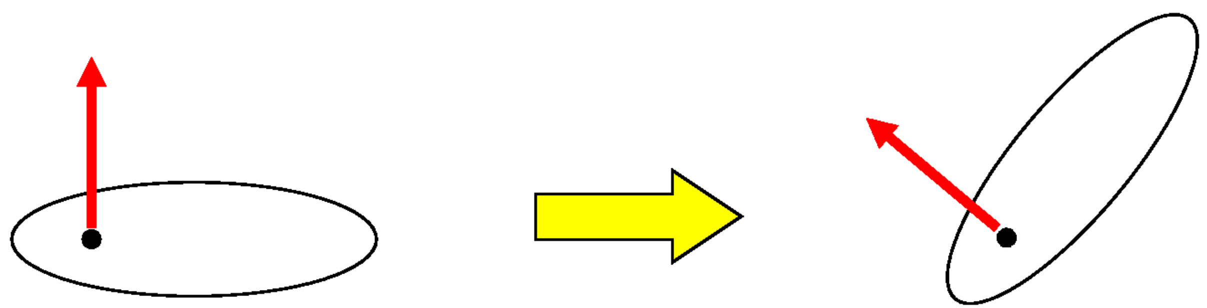
Stationary ellipses
in point mass potential



Planar rosettes in
spherical potential



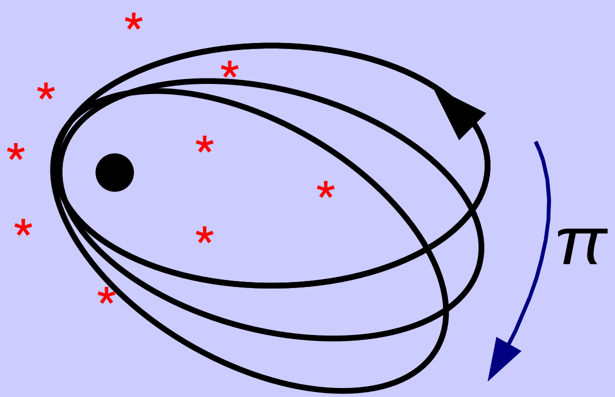
Scalar resonant relaxation

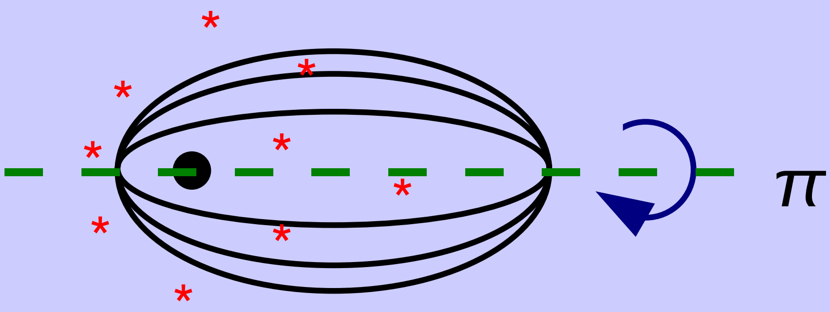


Vector resonant relaxation

Relevant timescales

What can be τ_ω ?





$$\tau_\omega = \tau_\phi = A_\phi \left(M / m\sqrt{N} \right)$$

$$\tau_\omega = \tau_M = A_M \left(M / Nm \right)$$

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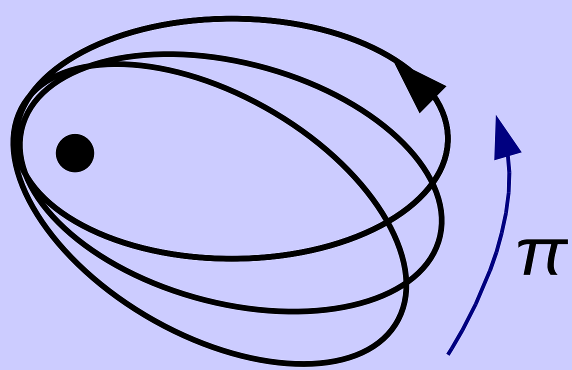
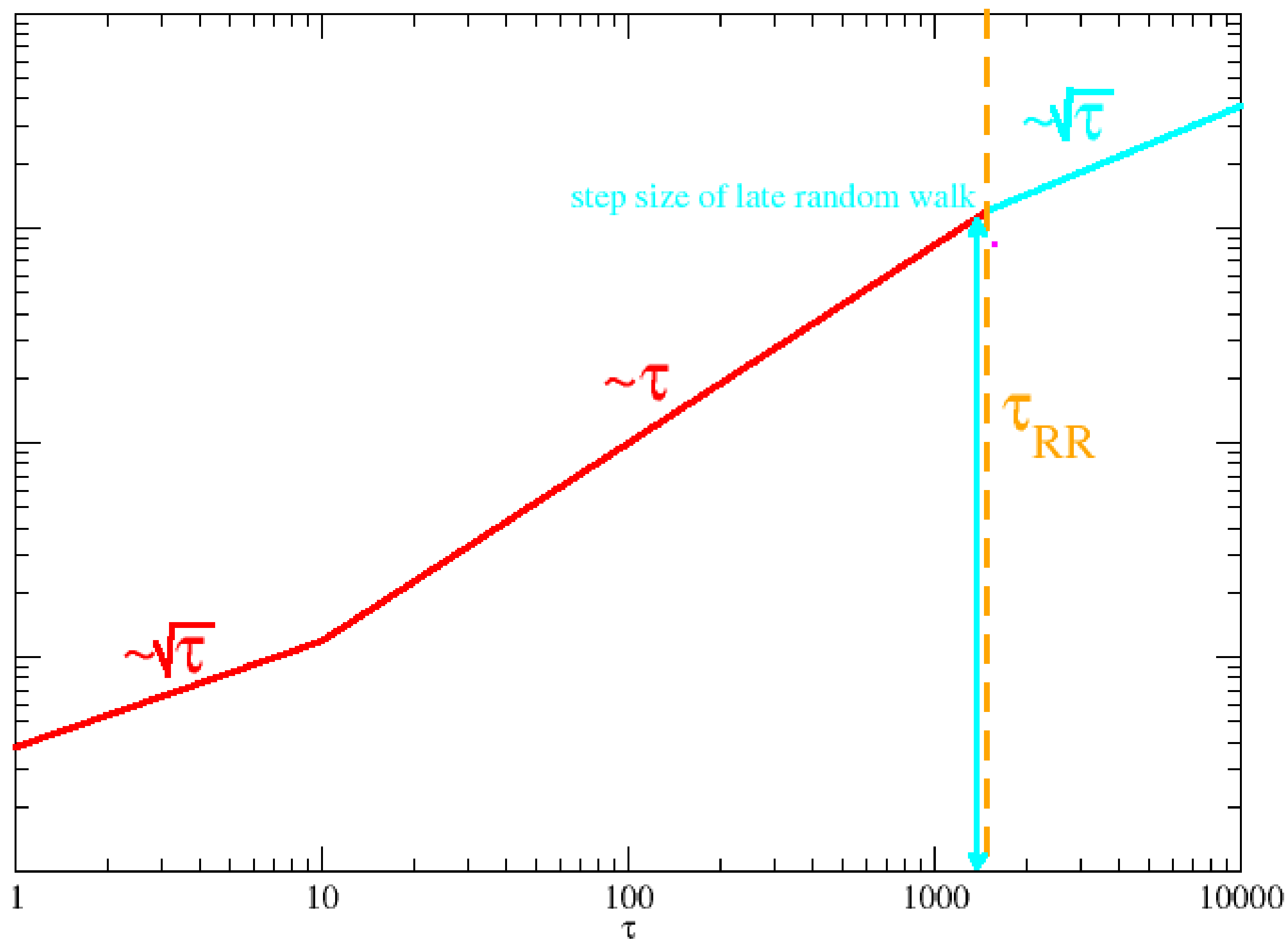
$$\tau_{1\mathrm{PN}} = \frac{a\left(1-e^2\right)}{6\mathrm{r}_g}$$



Schematic behavior of RR



Measured coefficients

NR

$$\left\langle \left(\left| E_2 - E_1 \right| / E_1 \right)^2 \right\rangle^{1/2} = \alpha_\Lambda \sqrt{N} (m/M) \sqrt{\tau}$$

$$\left\langle \left(\left| J_2 - J_1 \right| / J_{c,1} \right)^2 \right\rangle^{1/2} = \eta_{s\Lambda} \sqrt{N} (m/M) \sqrt{\tau}$$

$$\left\langle \left(\left| \mathbf{J}_2 - \mathbf{J}_1 \right| / J_{c,1} \right)^2 \right\rangle^{1/2} = \eta_{v\Lambda} \sqrt{N} (m/M) \sqrt{\tau}$$

RR

$$\left\langle \left(\left| J_2 - J_1 \right| / J_{c,1} \right)^2 \right\rangle^{1/2} = \beta_s \sqrt{N} (m/M) \tau$$

$$\left\langle \left(\left| \mathbf{J}_2 - \mathbf{J}_1 \right| / J_{c,1} \right)^2 \right\rangle^{1/2} = \beta_v \sqrt{N} (m/M) \tau$$

More than hundred simulations (*N*=200) with thermally distributed stars with varying massratio (*Q*=*M/m*) and varying cusp distribution $(\rho(r) \propto r^{-\gamma})$.

$$\left\langle \left(\left| J_2 - J_1 \right| / J_{c,1} \right)^2 \right\rangle^{1/2} = \sqrt{N} (m/M) \sqrt{\eta_{s\Lambda}^2 \tau + \beta_s^2 \tau^2}$$

$$\left\langle \left(\left| \boldsymbol{J}_2 - \boldsymbol{J}_1 \right| / J_{c,1} \right)^2 \right\rangle^{1/2} = \sqrt{N} (m/M) \sqrt{\eta_{v\Lambda}^2 \tau + \beta_v^2 \tau^2}$$

Fitted expressions

Measured coefficients

$$\langle \alpha \rangle = 2.82 \pm 0.05$$

$$\langle \eta_s \rangle = 1.01 \pm 0.03 \; \langle \eta_v \rangle = 1.64 \pm 0.04$$

$$\langle \beta_s \rangle = 1.05 \pm 0.02 \; \langle \beta_v \rangle = 1.83 \pm 0.03$$

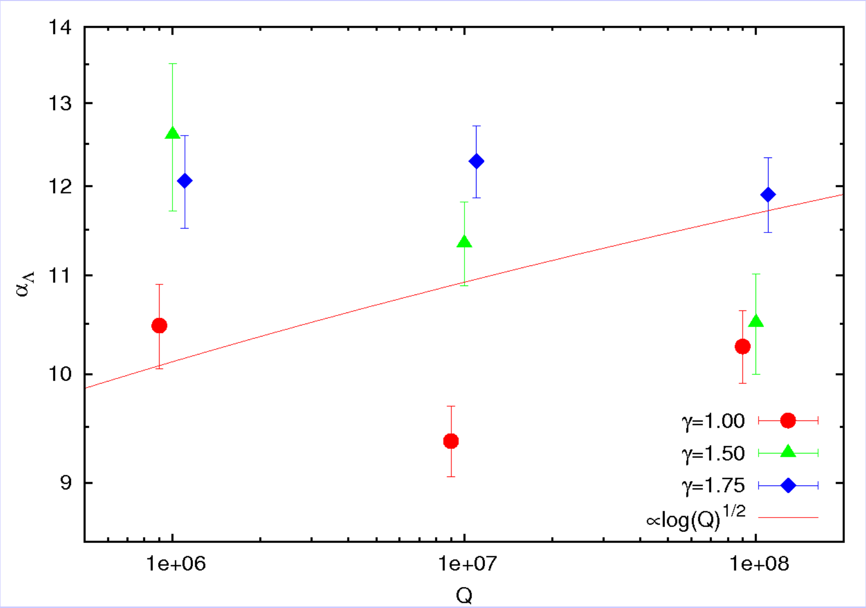
These are different from RT96 but good agreement with Gürkan & Hopman 2007

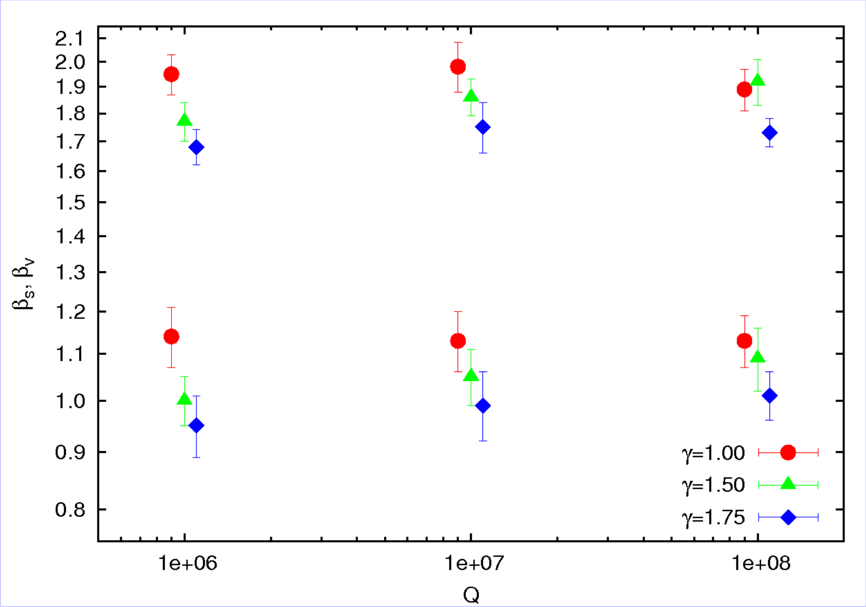
$$\langle \eta_v \rangle / \langle \eta_s \rangle = 1.61 \pm 0.06$$ NR

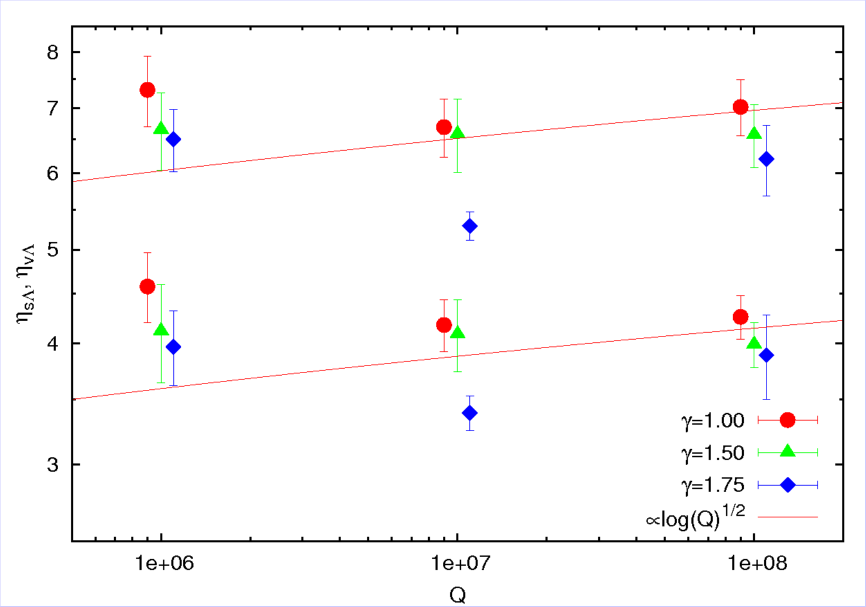
$$\langle \beta_v \rangle / \langle \beta_s \rangle = 1.74 \pm 0.04$$ RR

Eilon, Kupi, & Alexander 2009

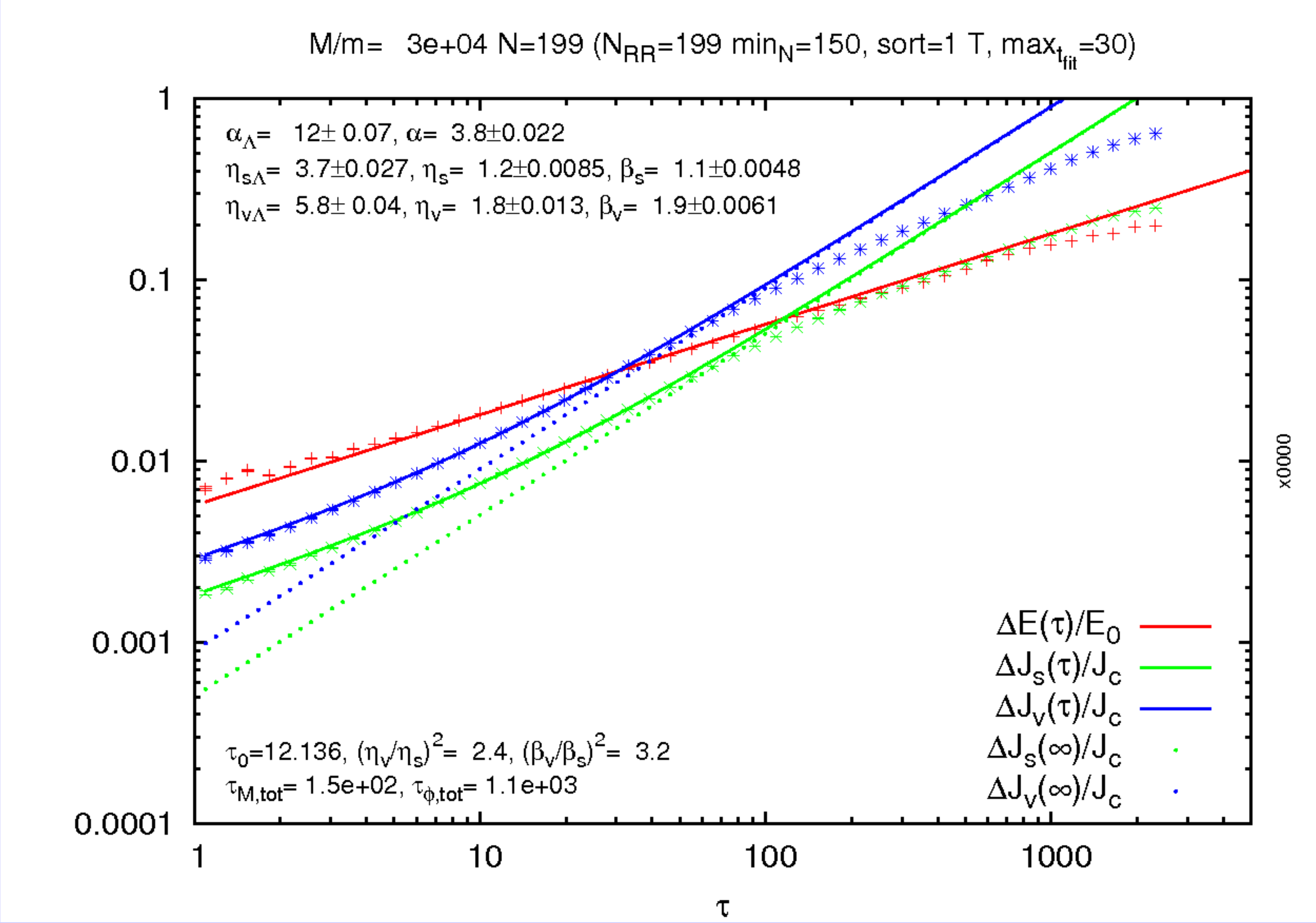
Measured coefficients of NR





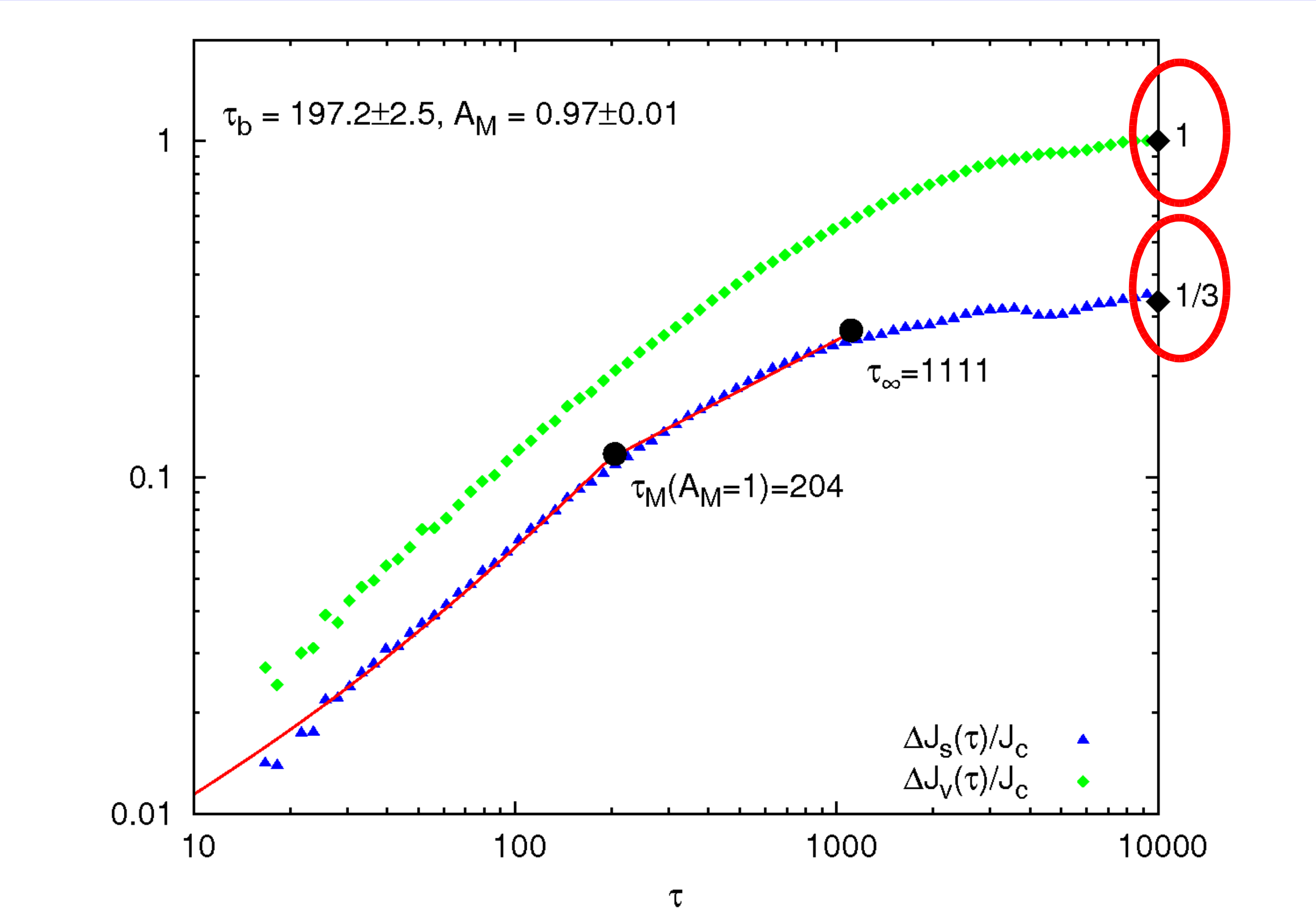


Long term evolution of correlation curves



$N = 200, \gamma = 1, Q = 3 \cdot 10^4$

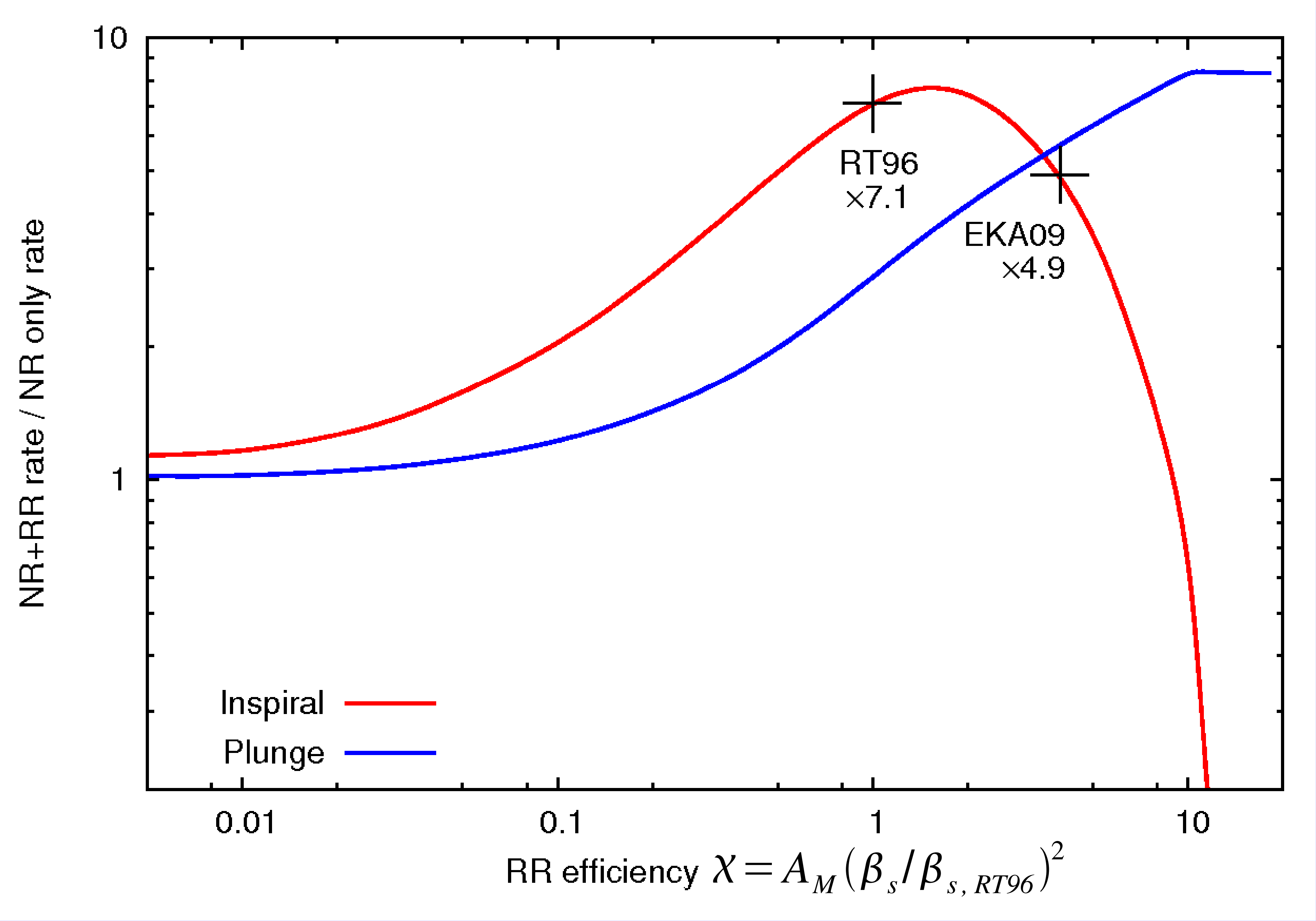
Long term evolution of correlation curves



$N=50, \gamma=1, Q=10^4$

$$\tau_M = A_M \frac{M}{Nm}$$ measured $$\Rightarrow A_M = 0.99 \pm 0.06$$

Implications of RR for EMRI rates

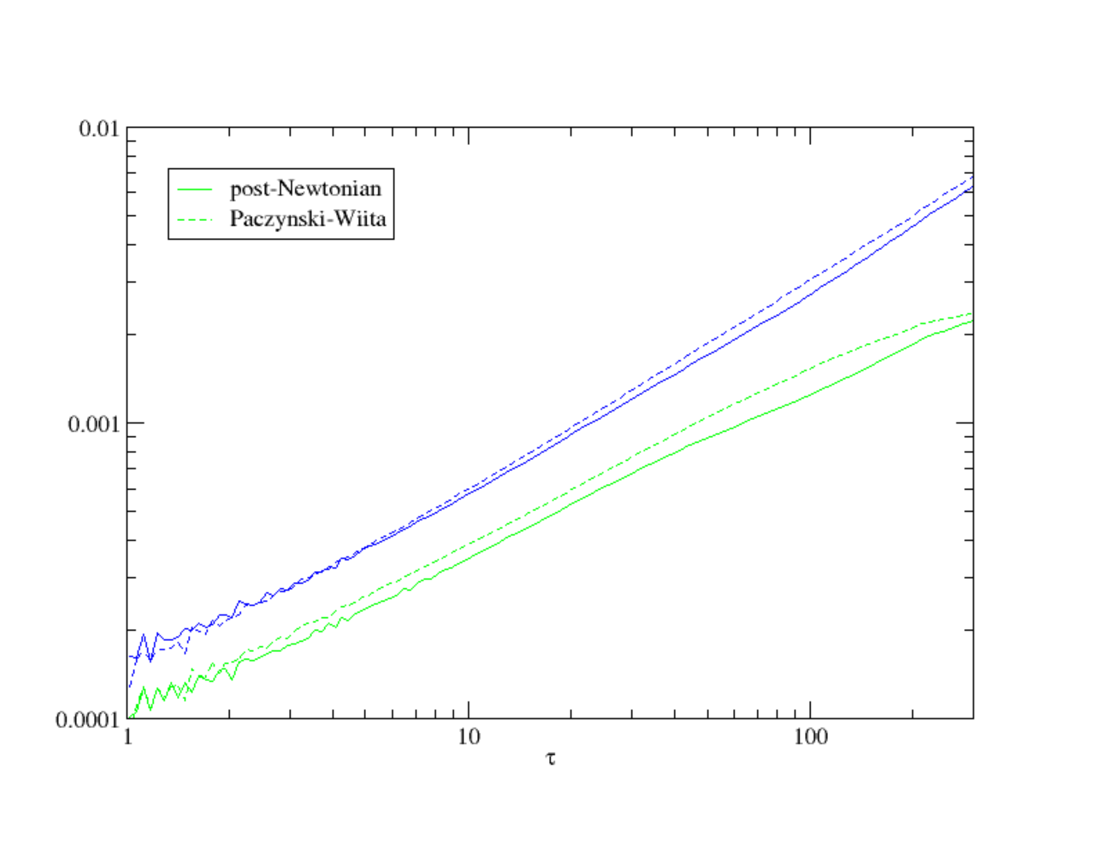


Paczynski-Wiita potential instead of GR (full or PN)

$$\phi(r) = -\frac{GM}{r - r_G}, r_G = \frac{2GM}{c^2}$$

ISCO and marginally bound orbit are the same as in GR $(3r_G, 2r_G)$

Precession is 1.4-1.5 faster than in 2PN case.



RR in case of GR

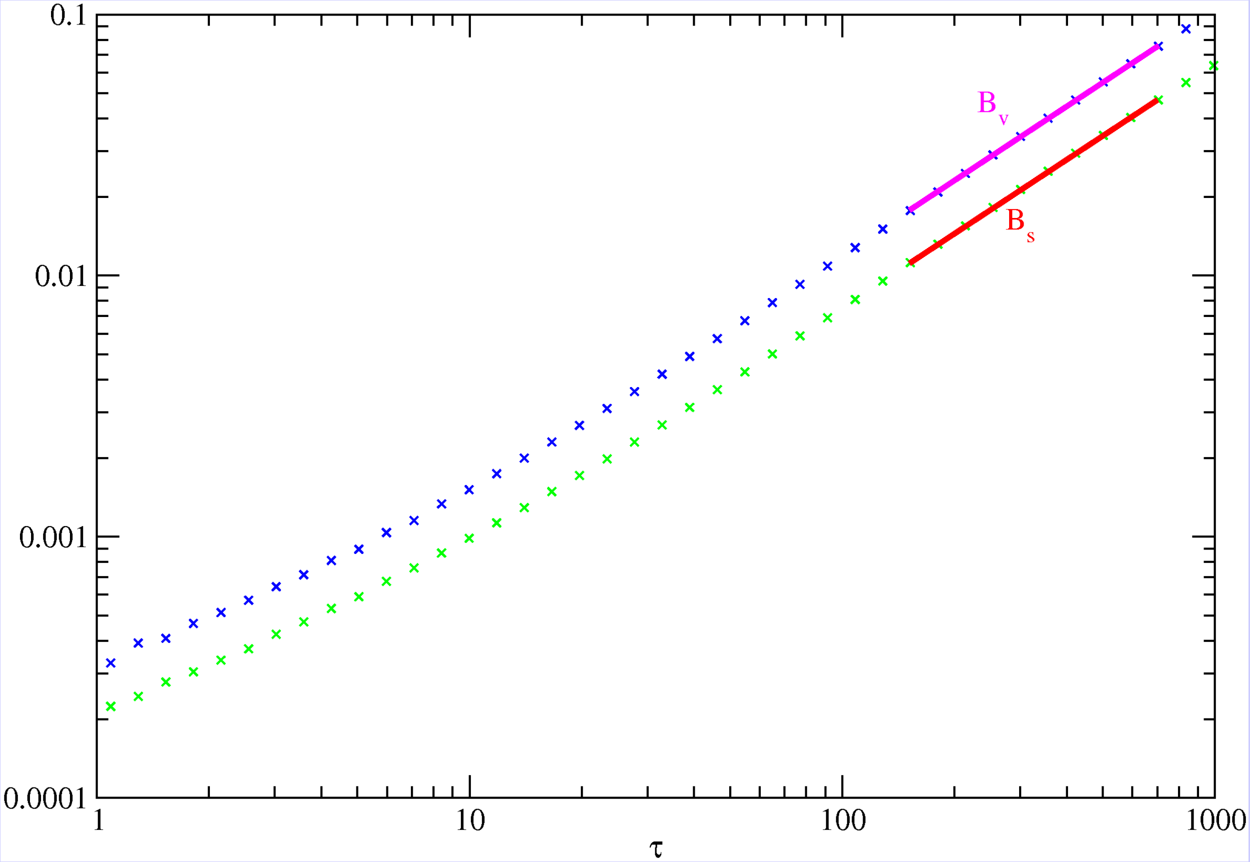
If there is precession, RR is quenched. It happens after $\tau_\omega < \tau$.
In the case of relativistic precession the timescale of scalar RR is

$$T^s_{RR} \propto \left| \frac{1}{t_M} - \frac{1}{t_{GR}} \right|$$

How can we measure quenching?

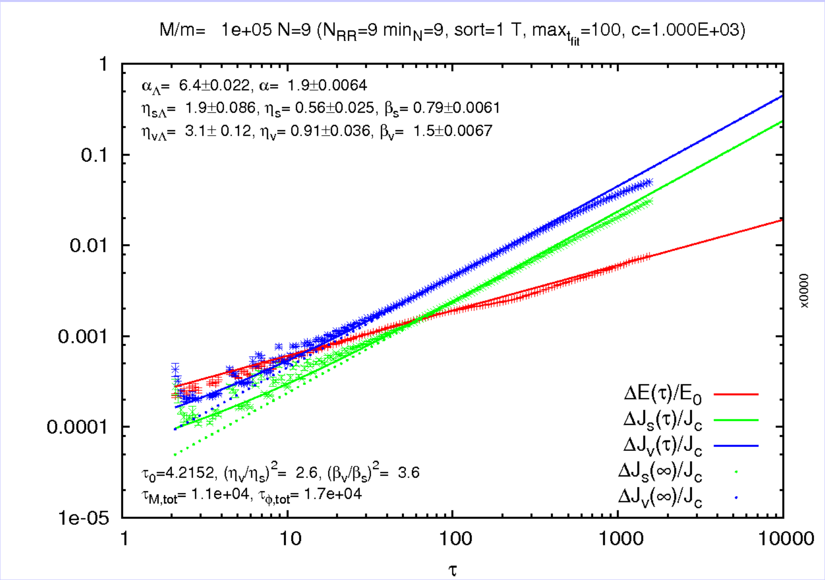
B~1 => there is RR
B~0.5 => no RR

$$\Delta J / J_c \propto \tau^B$$



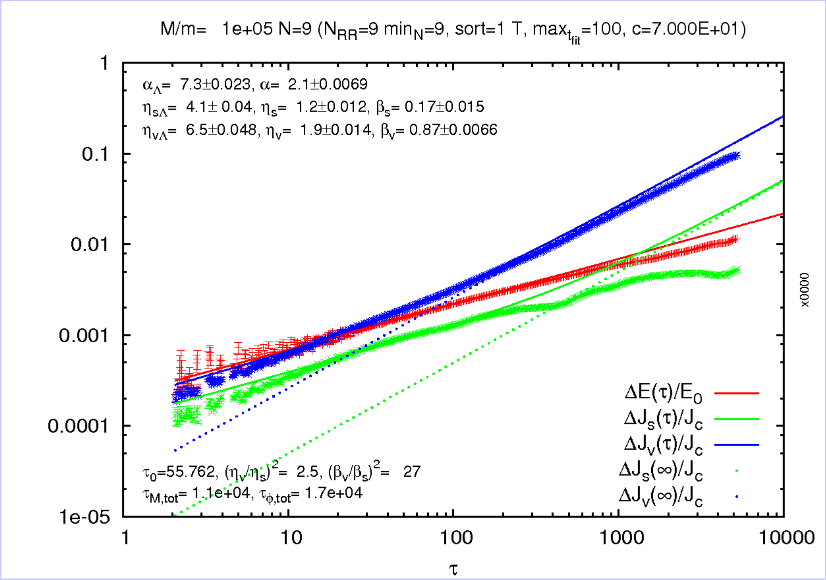
RR in case of GR

PW



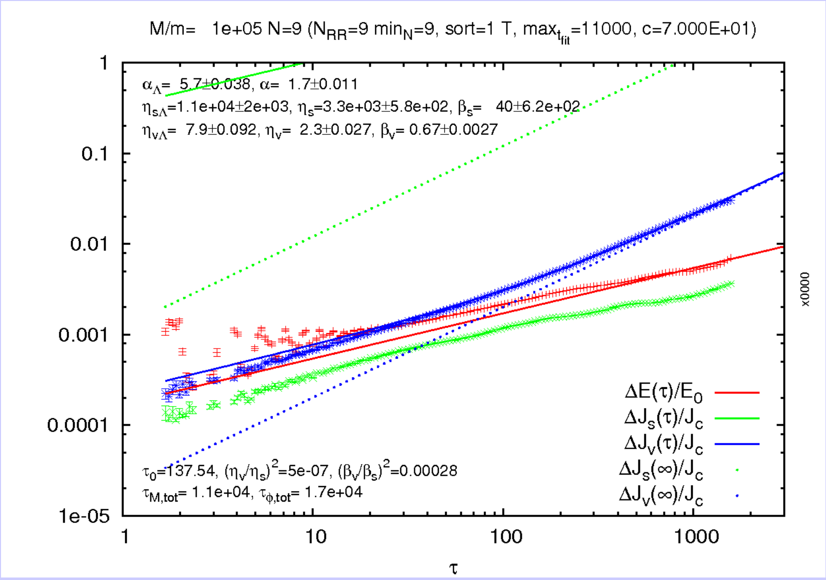
$$B_s = 0.91\,,\, B_v = 0.84, \tau_{GR} = ۲.۷ \cdot 10^4$$

PW



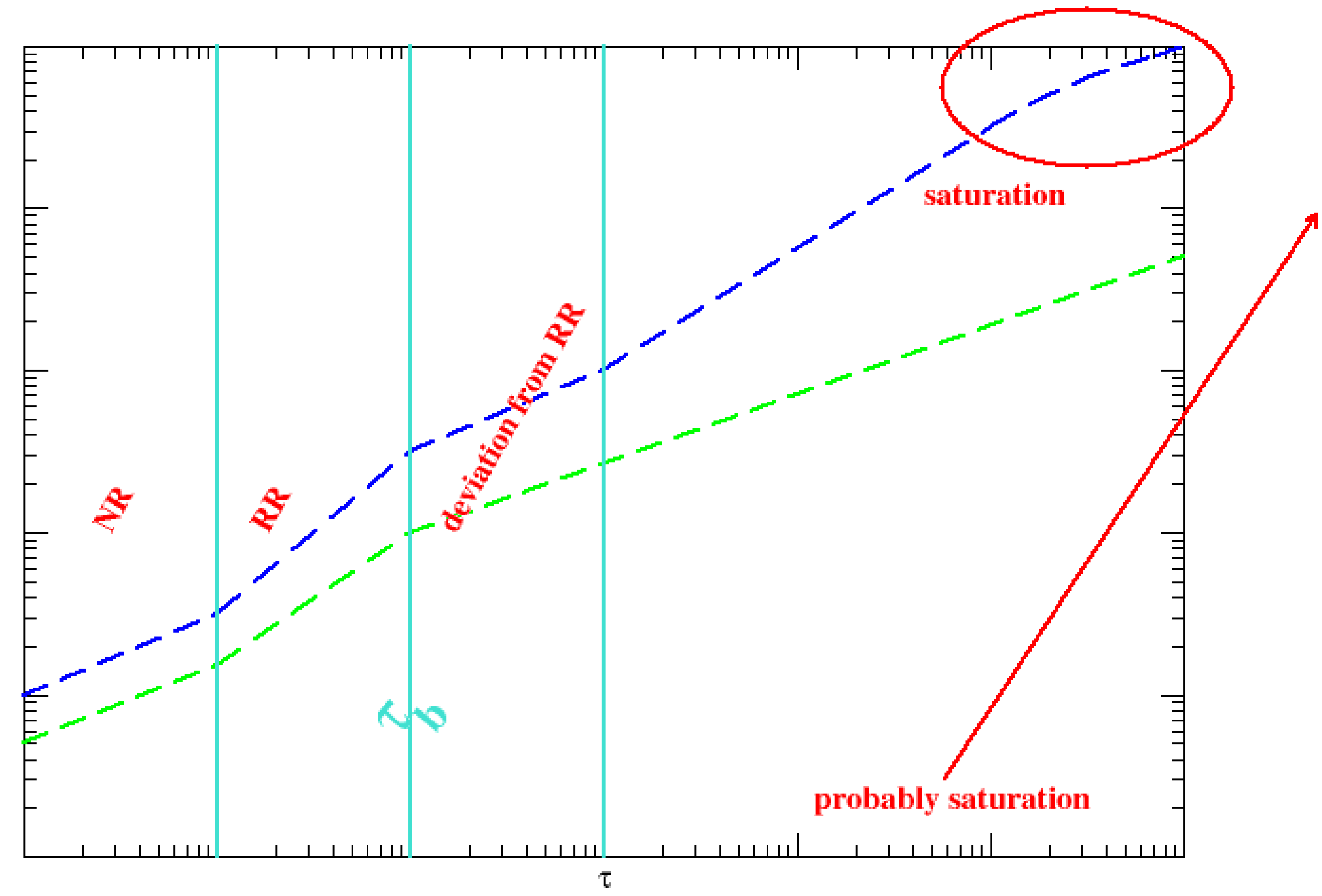
$$B_s = 0.55\,,\, B_v = ۰.۹۵, \tau_{GR} = 136$$

PN



$$B_s = 0.34\,,\, B_v = 0.89, \tau_{GR} = 136$$

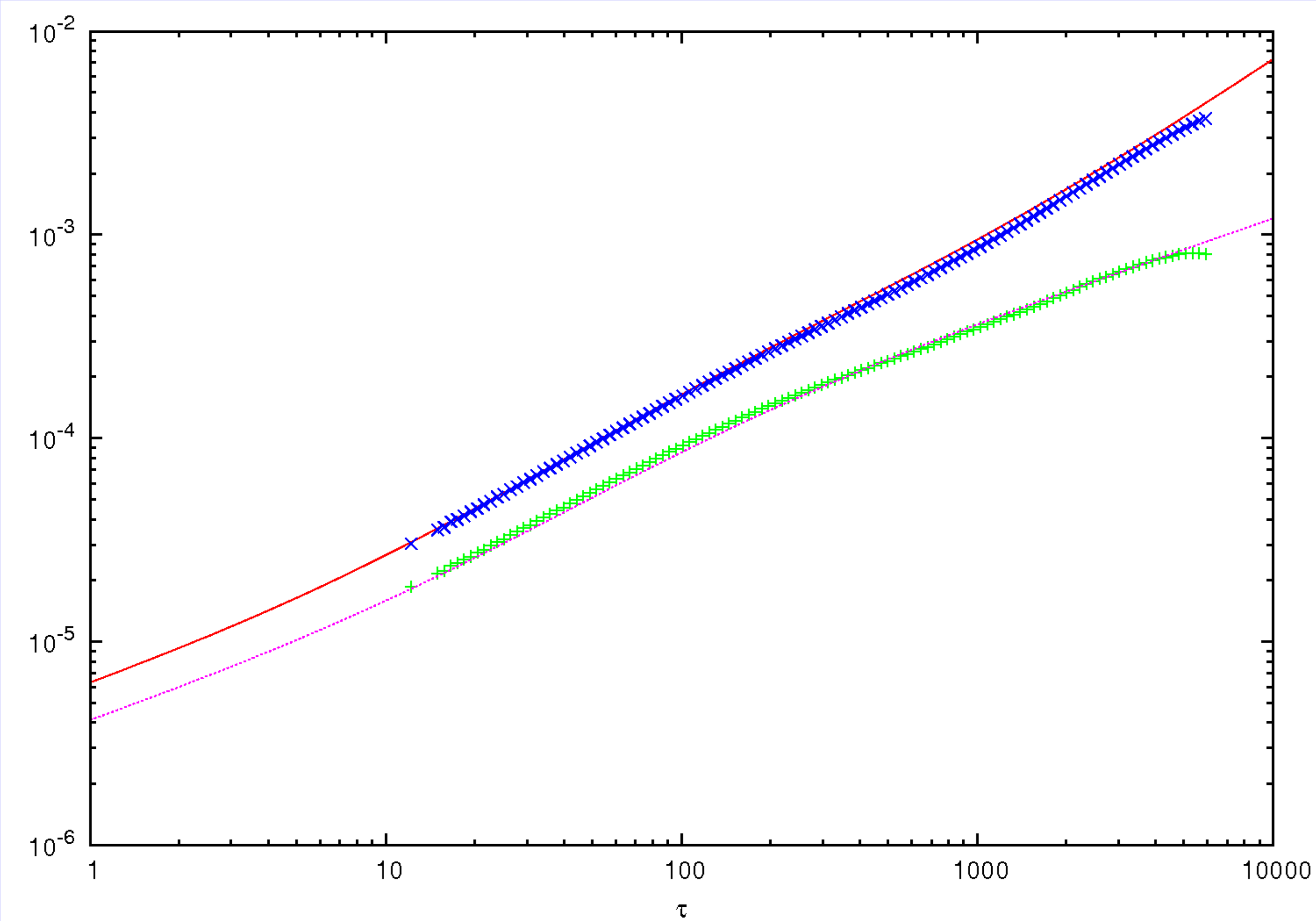
RR in case of GR



$$S(\tau) \propto \sqrt{\frac{1}{\frac{\tau_b}{\tau} + \left(\frac{\tau_b}{\tau}\right)^2}}$$

$$\delta \boldsymbol{J} = \frac{\sqrt{N}}{Q} \sqrt{\eta_{s\Lambda}^2 \tau + \frac{\beta_s^2 \tau_b^2}{\frac{\tau_b}{\tau} + \left(\frac{\tau_b}{\tau}\right)^2} + C^2 \tau^2}$$

$$\delta J = \frac{\sqrt{N}}{Q} \sqrt{\eta_{s\Lambda}^2 \tau + \frac{\beta_s^2 \tau_b^2}{\frac{\tau_b}{\tau} + \left(\frac{\tau_b}{\tau}\right)^2}}$$



C, τ_b have to be measured.

Summary

- New measurement of RR coefficients
- We studied the long term evolution of correlation curves
- Implications of RR for EMRI rates: higher than pure NR gives
- Preliminary relativistic results