Parameter estimation errors for radiating binaries with non-trivial mass ratios

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in collaboration with

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Stars and Singularities
Weizmann Institute of Science, Rehovot, Israel
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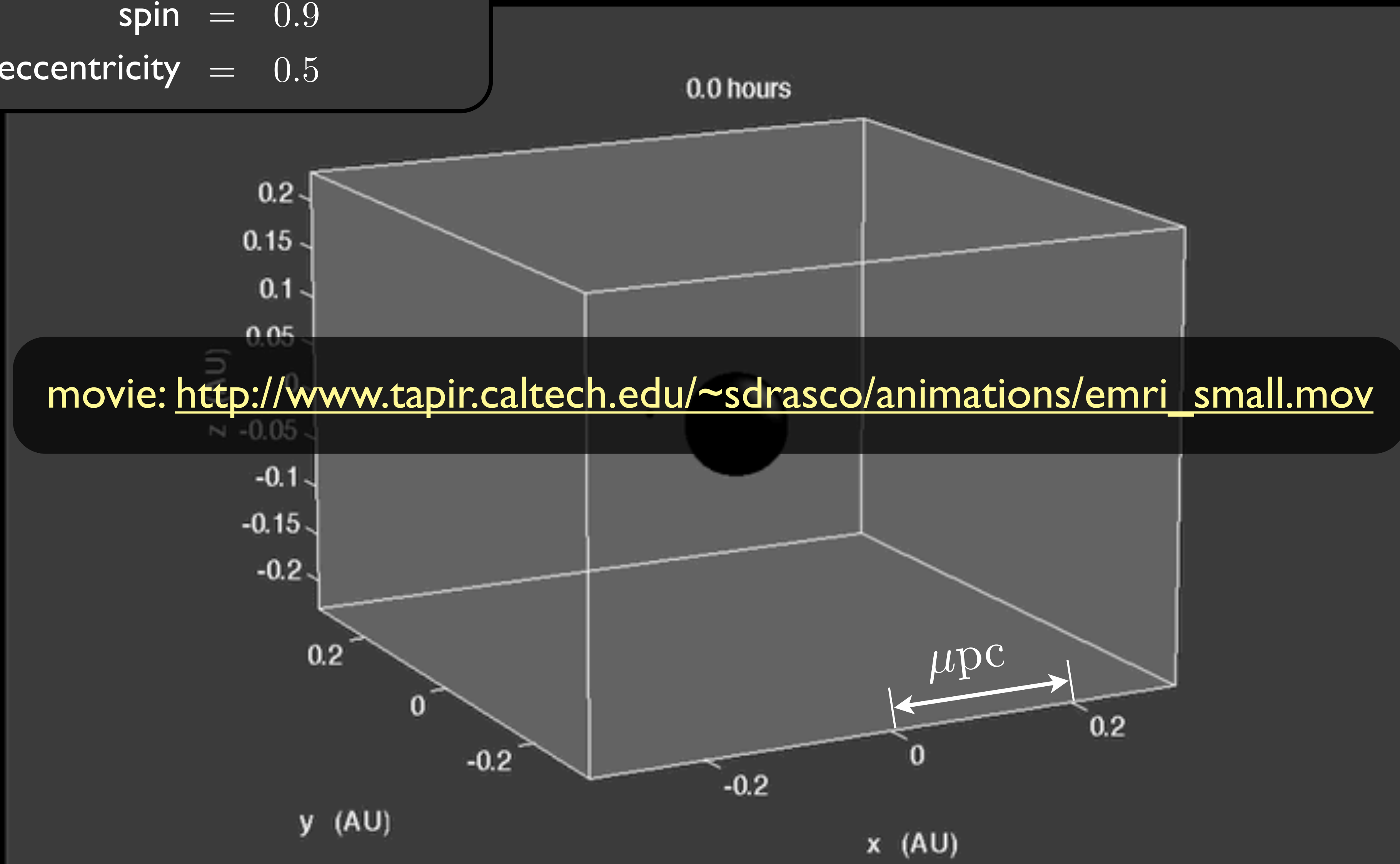
Setting the scale

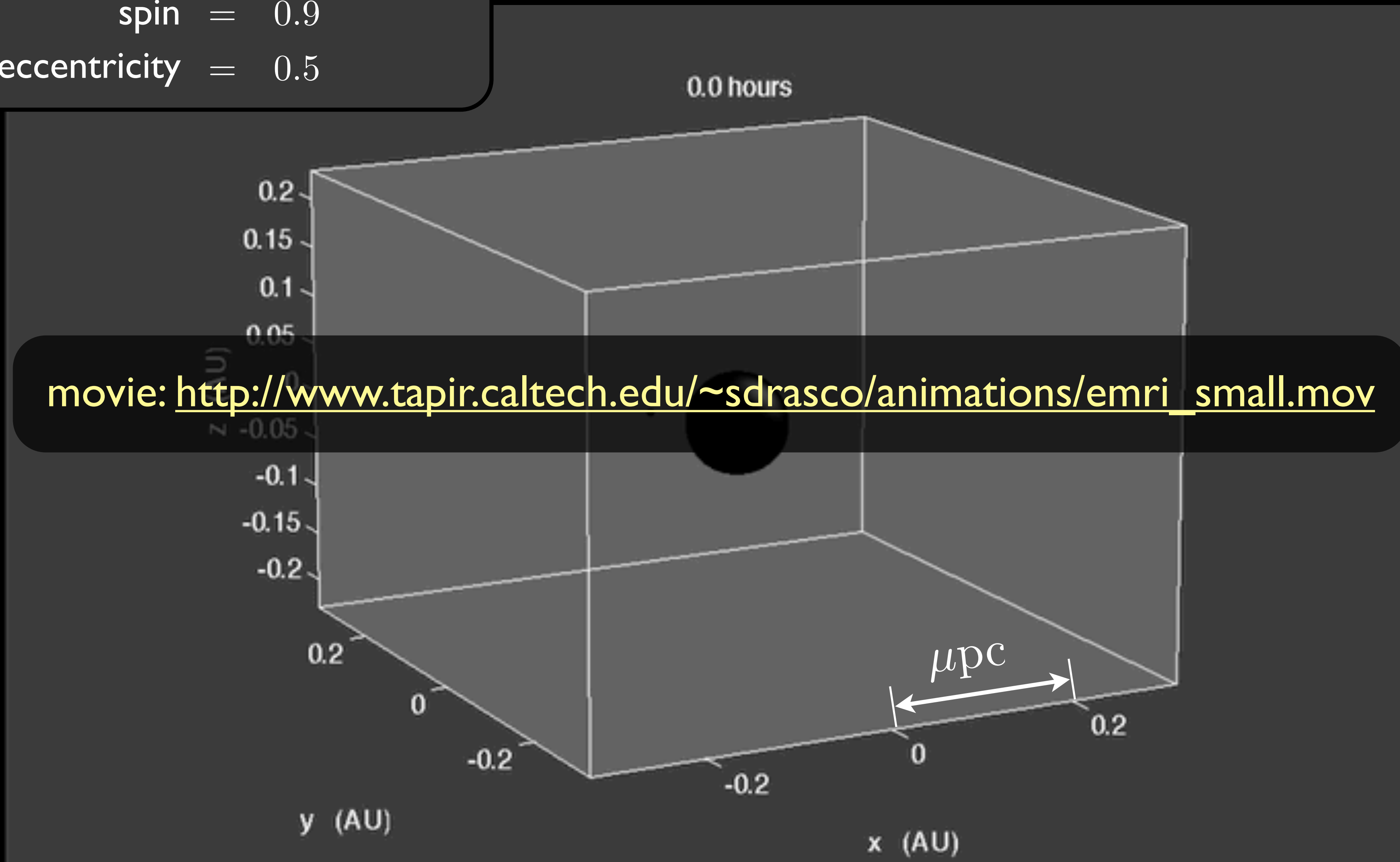
$$\begin{aligned} \text{mass} &= 3.5 \times 10^6 M_\odot \\ \text{semi-major axis} &= 6\ M \\ \text{spin} &= 0.9 \\ \text{eccentricity} &= 0.5 \end{aligned}$$



movie: http://www.tapir.caltech.edu/~sdrasco/animations/emri_small.mov

mass	=	$3.5 \times 10^6 M_\odot$
semi-major axis	=	$6\ M$
spin	=	0.9
eccentricity	=	0.5

For LISA: $10^5 \lesssim M/M_\odot \lesssim 10^7$



movie: http://www.tapir.caltech.edu/~sdrasco/animations/emri_small.mov

mass	$=$	$3.5 \times 10^6 M_\odot$
semi-major axis	$=$	$6\ M$
spin	$=$	0.9
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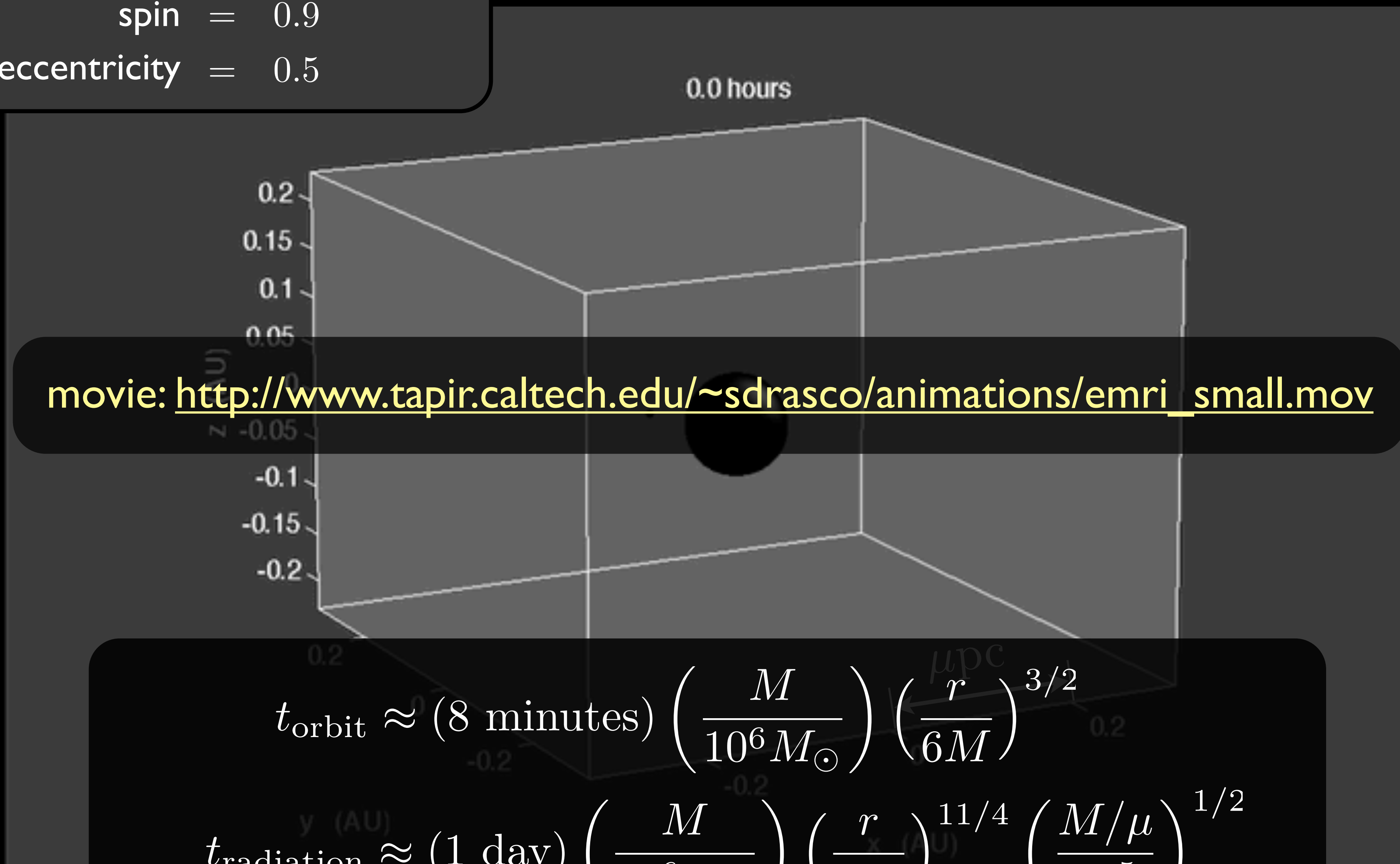


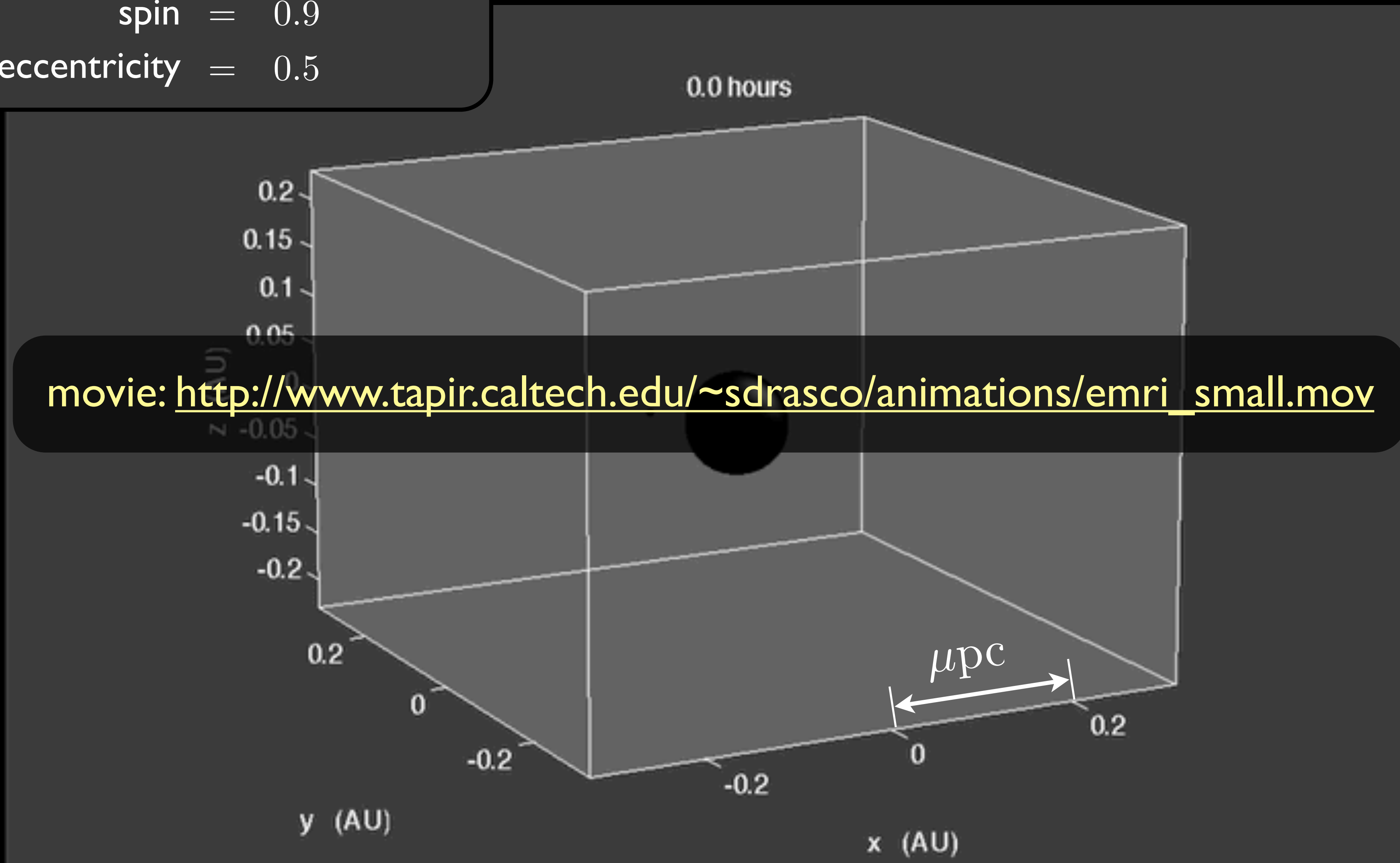
movie: http://www.tapir.caltech.edu/~sdrasco/animations/emri_small.mov

$$t_{\rm orbit} \approx (8\ \text{minutes}) \left(\frac{M}{10^6 M_\odot}\right) \left(\frac{r}{6M}\right)^{3/2}$$

$$t_{\rm radiation} \approx (1\ \text{day}) \left(\frac{M}{10^6 M_\odot}\right) \left(\frac{r}{6M}\right)^{11/4} \left(\frac{M/\mu}{10^5}\right)^{1/2}$$

mass	$=$	$3.5 \times 10^6 M_\odot$
semi-major axis	$=$	$6\ M$
spin	$=$	0.9
eccentricity	$=$	0.5

For LIGO: $10 \lesssim M/M_\odot \lesssim 10^3$



movie: http://www.tapir.caltech.edu/~sdrasco/animations/emri_small.mov

mass	$=$	$3.5 \times 10^6 M_\odot$
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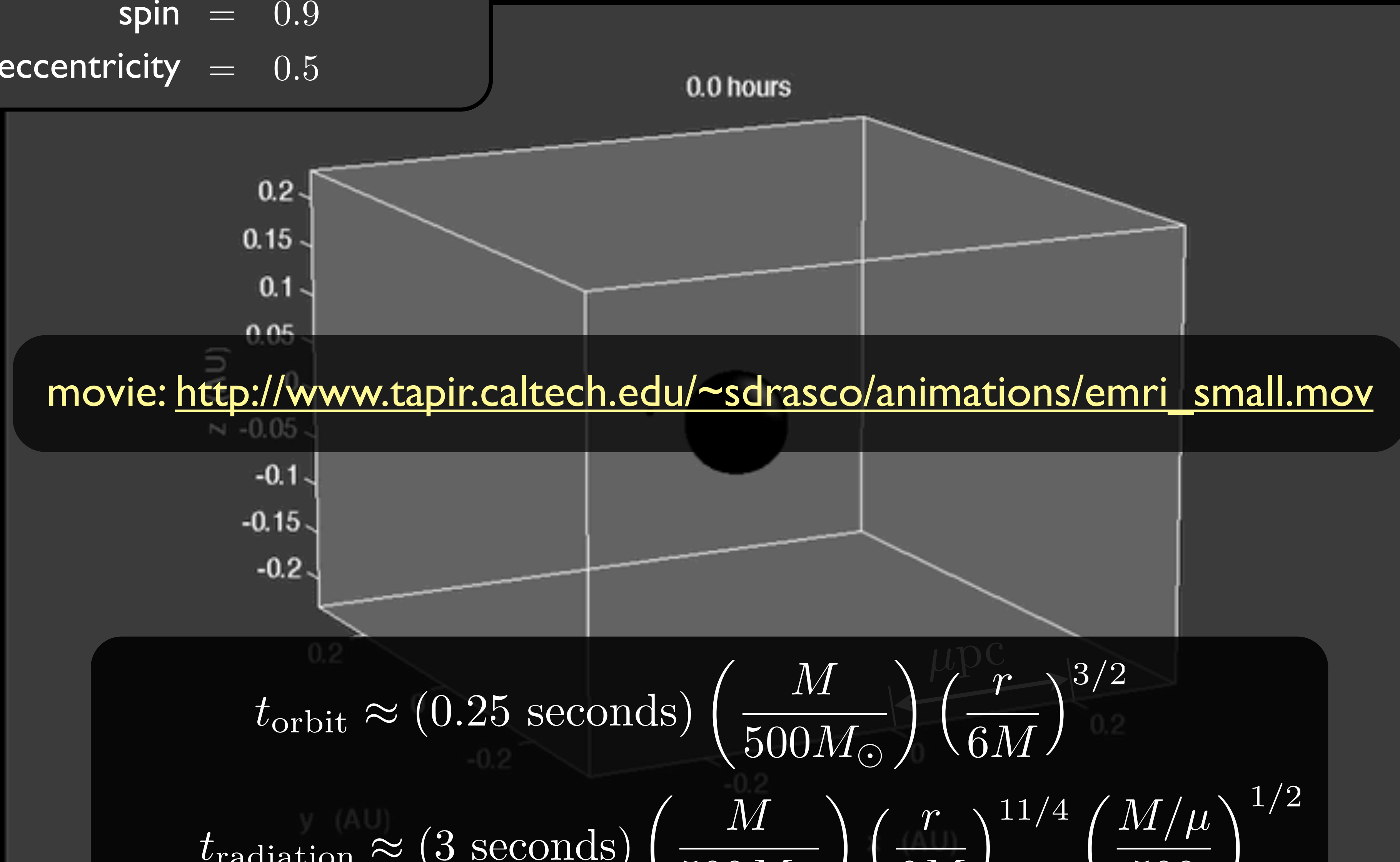


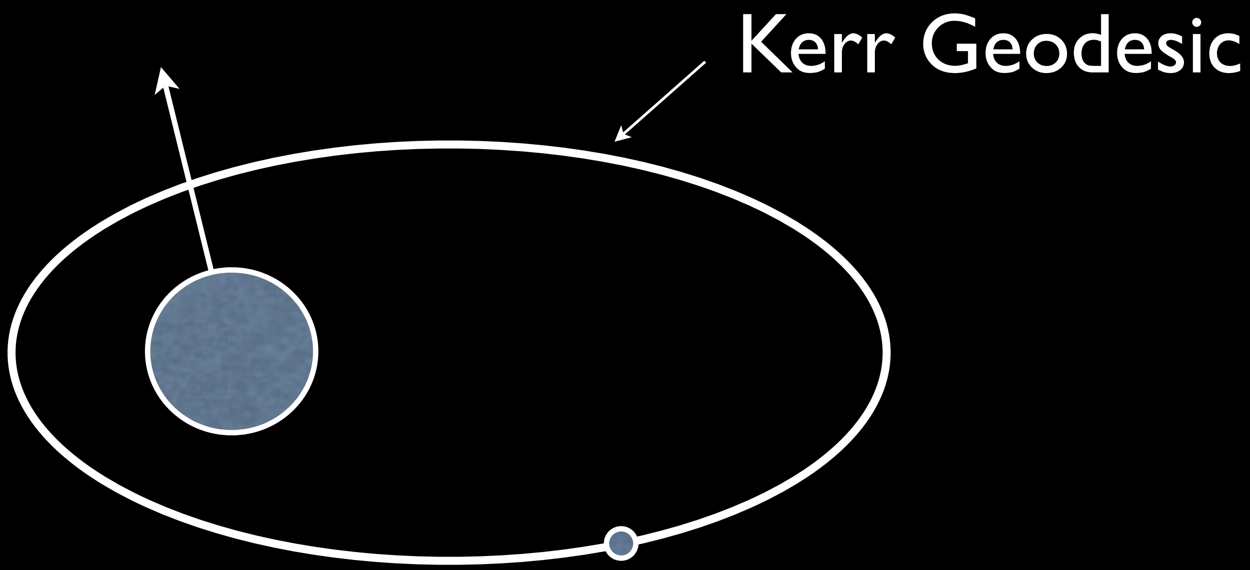
movie: http://www.tapir.caltech.edu/~sdrasco/animations/emri_small.mov

$$t_{\rm orbit} \approx (0.25\ \text{seconds}) \left(\frac{M}{500 M_\odot}\right) \left(\frac{r}{6M}\right)^{3/2}$$

$$t_{\rm radiation} \approx (3\ \text{seconds}) \left(\frac{M}{500 M_\odot}\right) \left(\frac{r}{6M}\right)^{11/4} \left(\frac{M/\mu}{500}\right)^{1/2}$$

Adiabatic inspiral

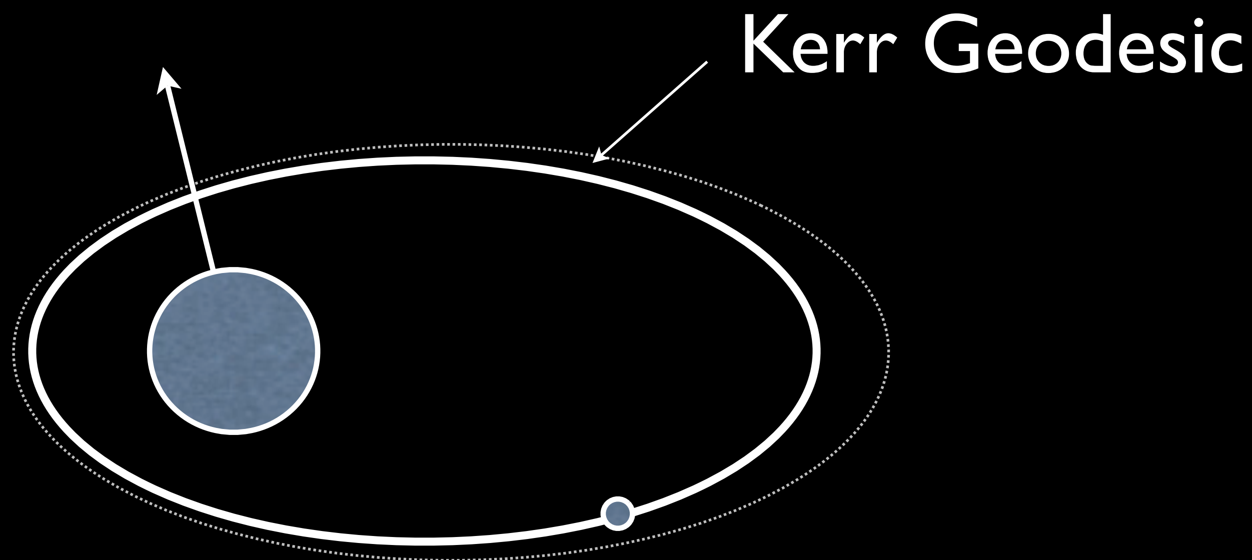
$$T_{\text{radiation}} \gg T_{\text{orbit}}$$



$$t = 0$$

Adiabatic inspiral

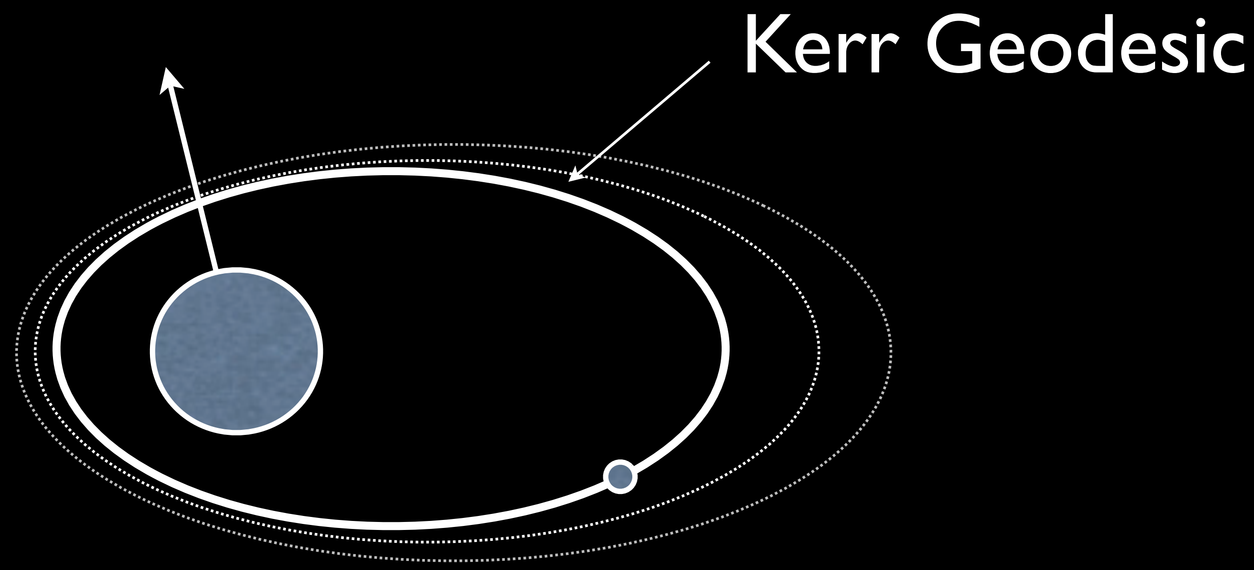
$$T_{\text{radiation}} \gg T_{\text{orbit}}$$



$$t = T_{\text{radiation}}$$

Adiabatic inspiral

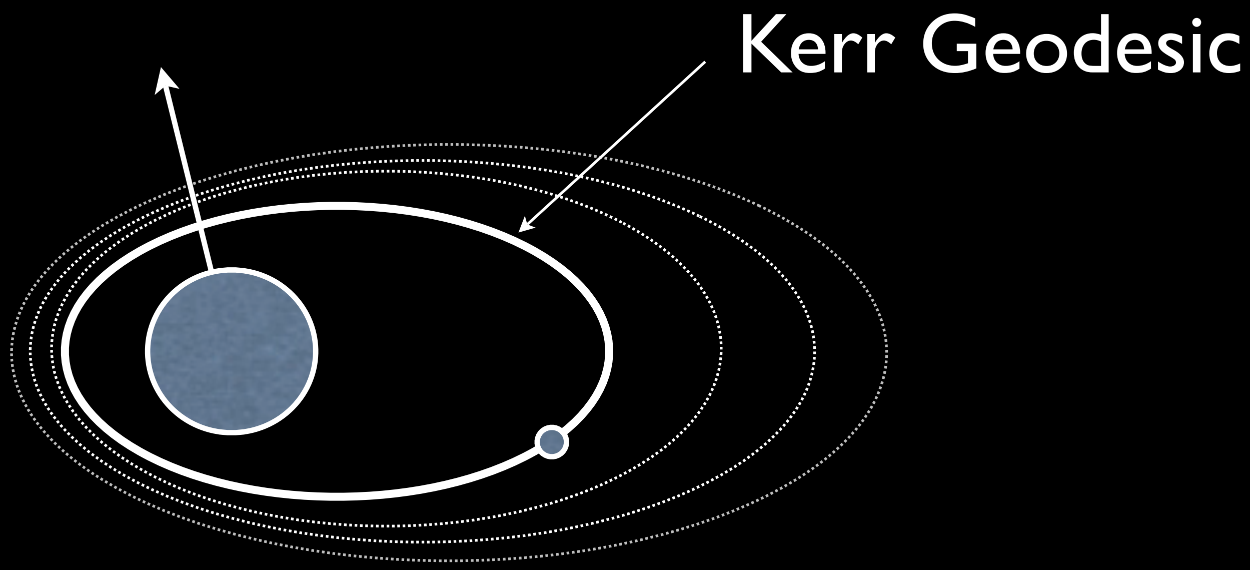
$$T_{\text{radiation}} \gg T_{\text{orbit}}$$



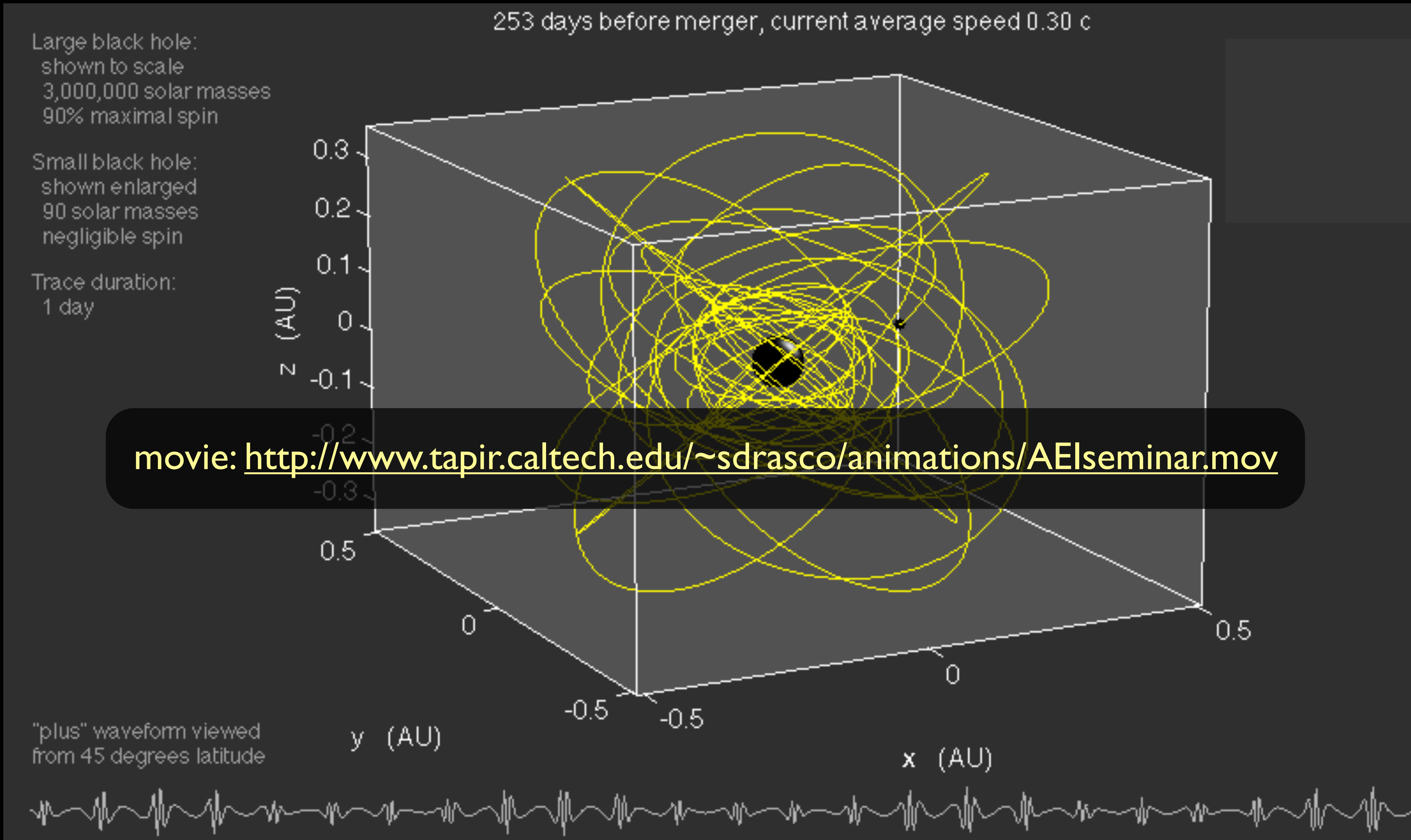
$$t = 2T_{\text{radiation}}$$

Adiabatic inspiral

$$T_{\text{radiation}} \gg T_{\text{orbit}}$$

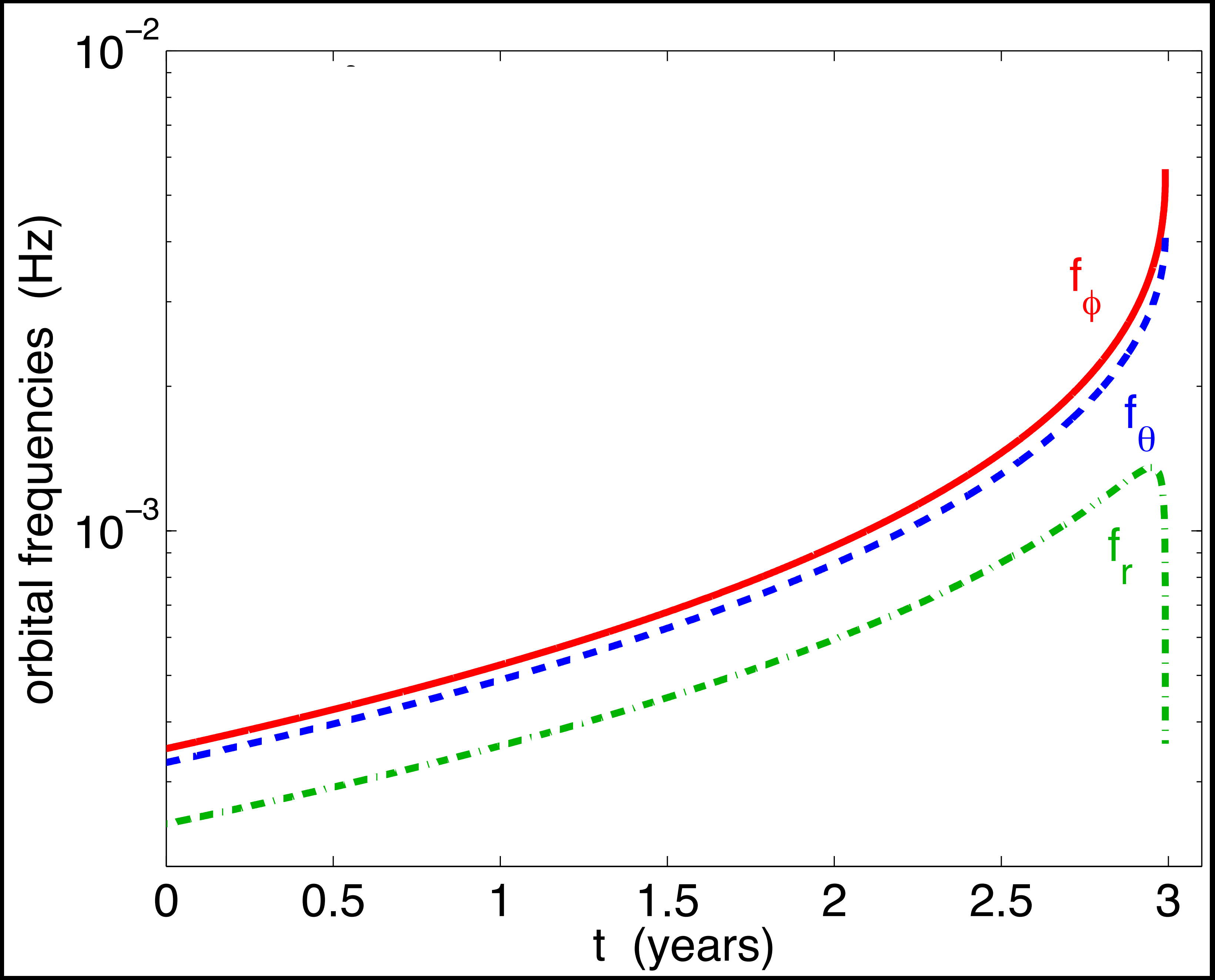


$$t = 3T_{\text{radiation}}$$



movie: http://www.tapir.caltech.edu/~sdrasco/animations/AEIseminar.mov

targets for observation: resonant orbits



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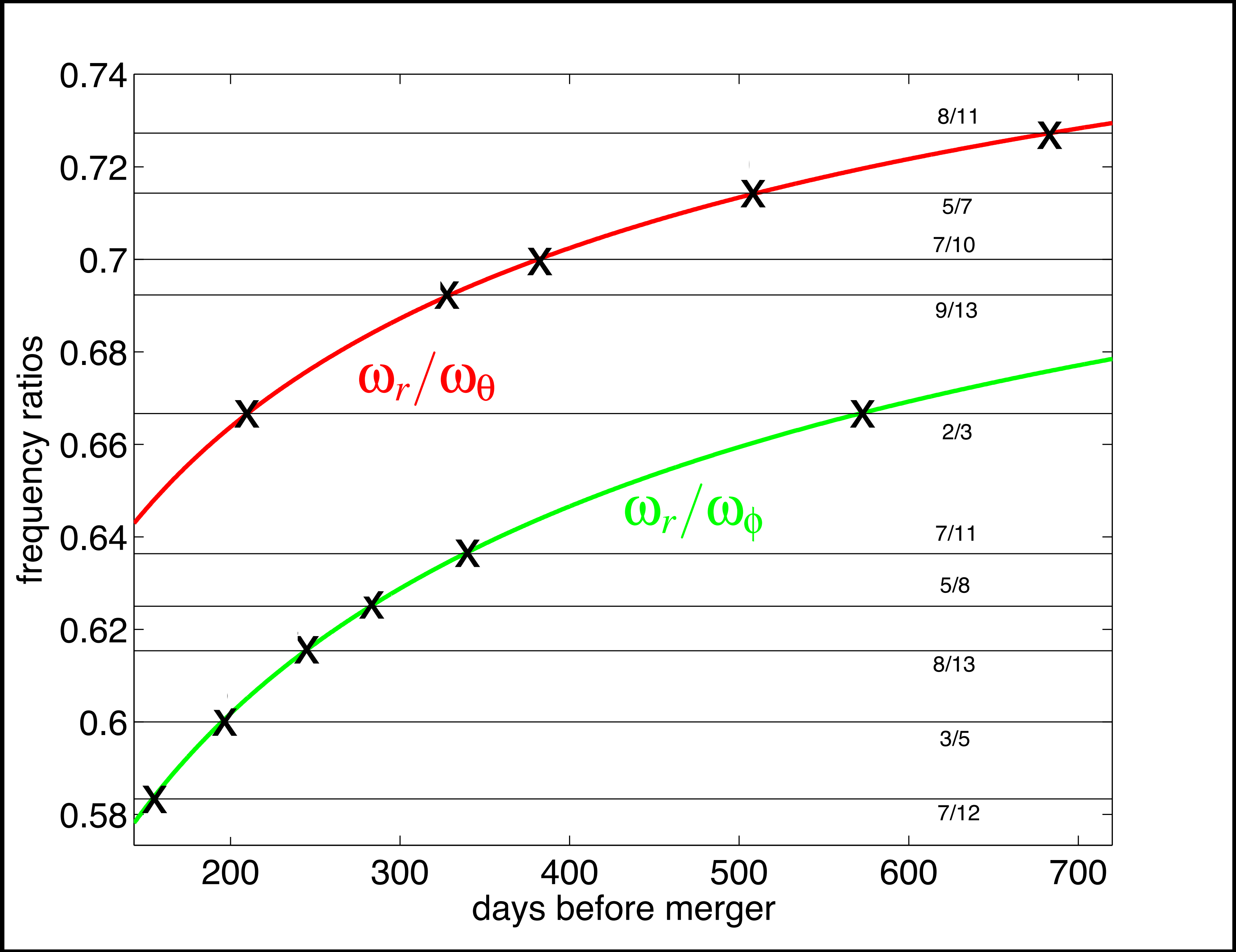
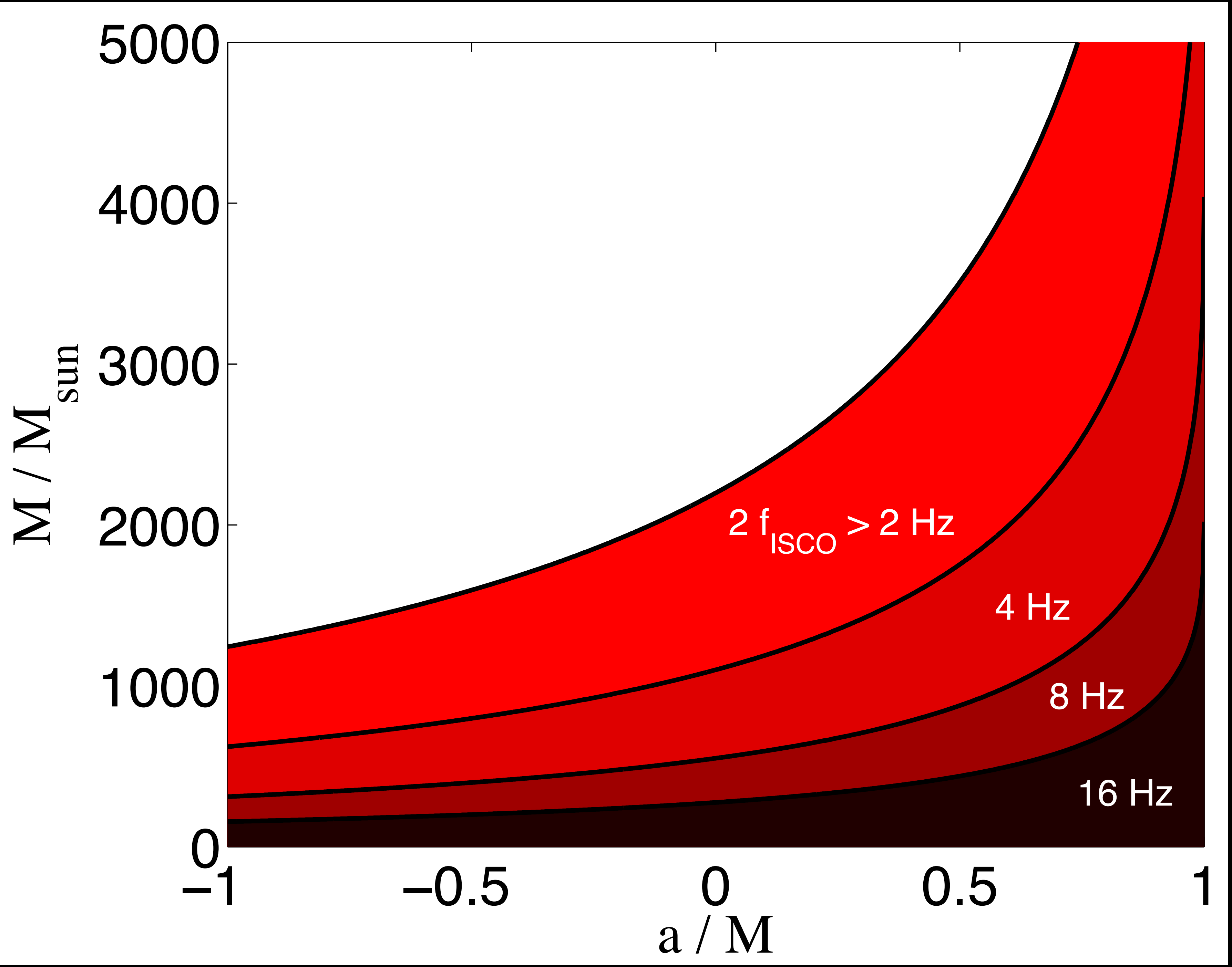


What can LIGO see: mass & spin targets



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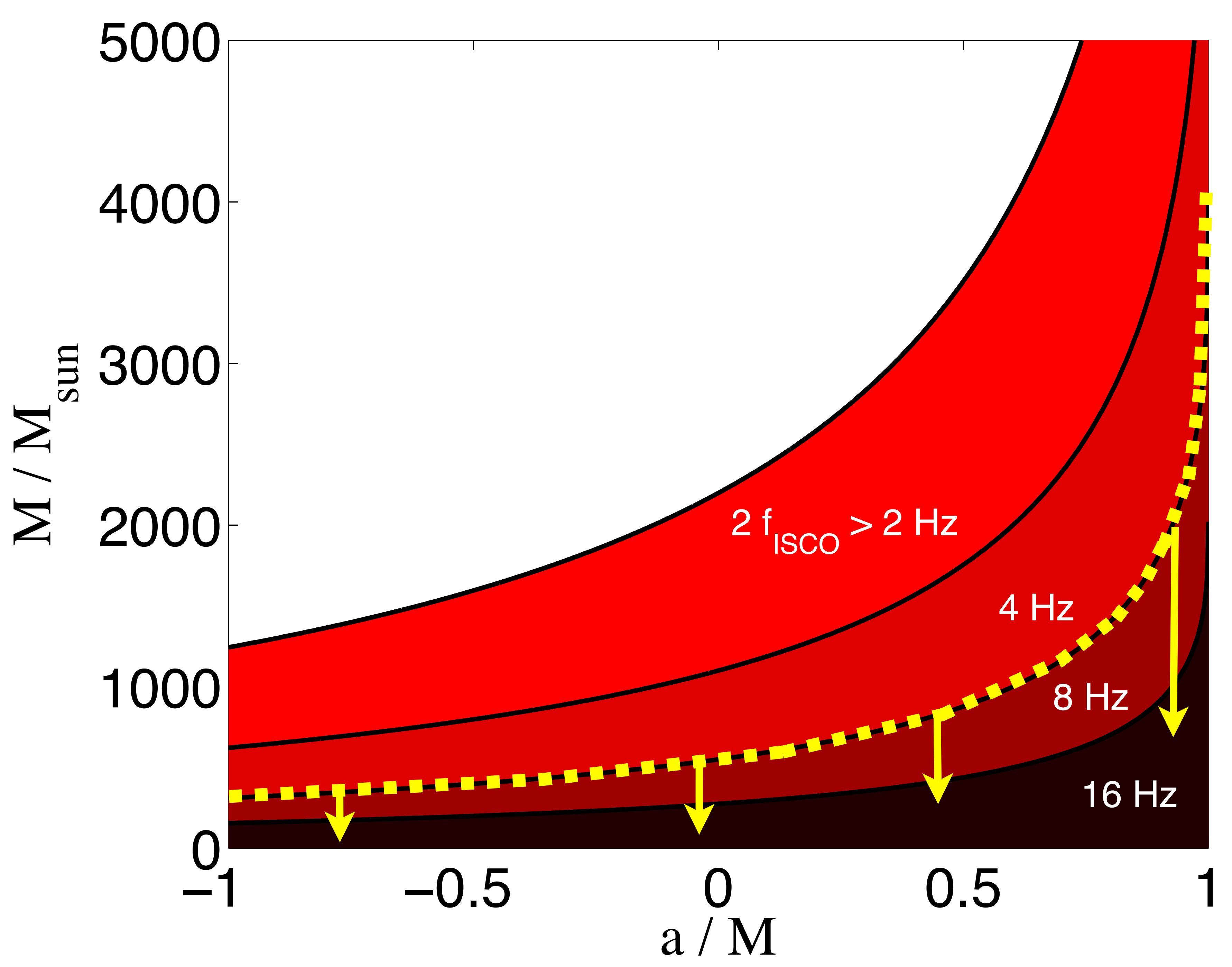
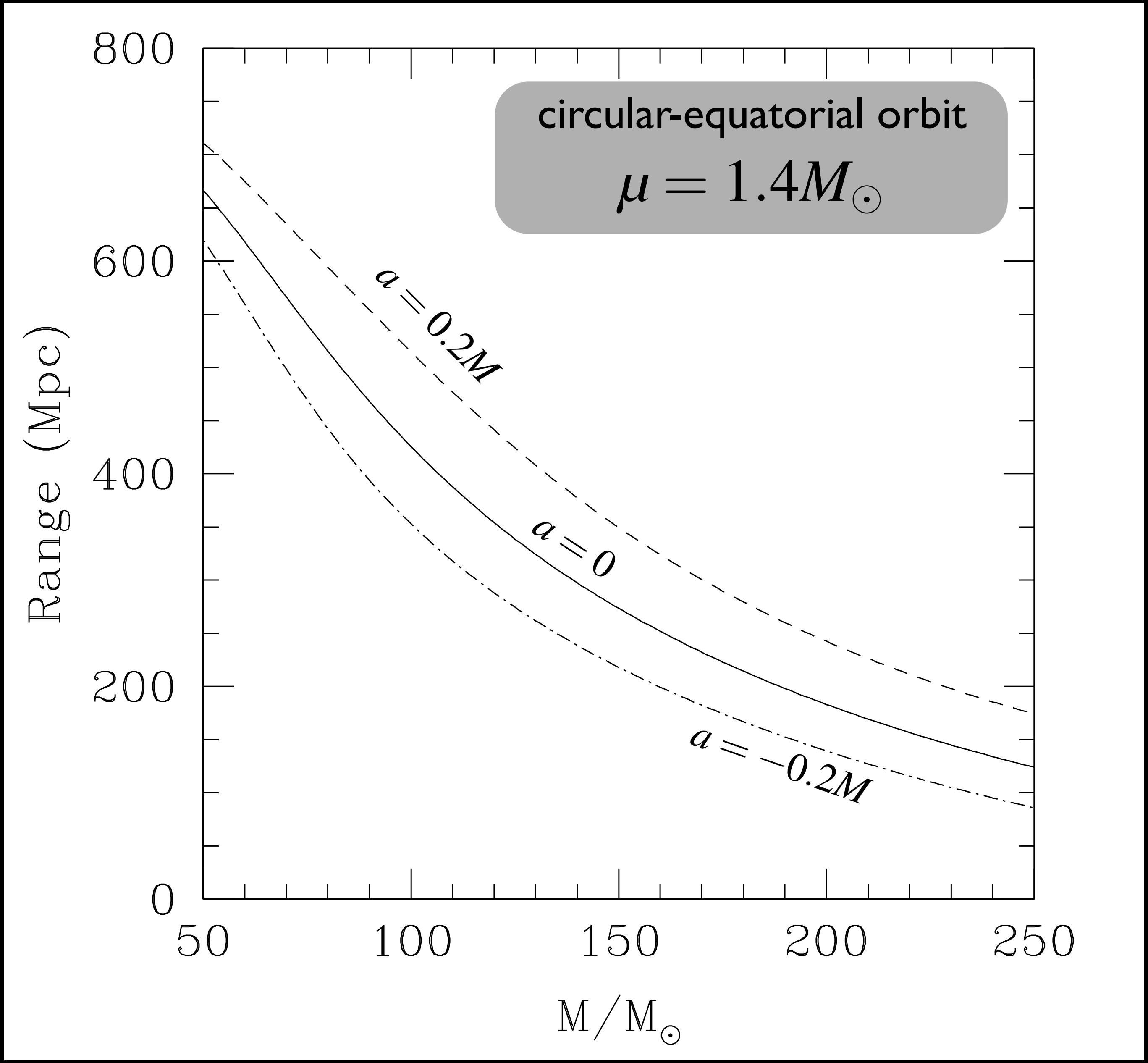


What can LIGO see: advanced LIGO range



from Mandel, Brown, Gair, Miller, 2008

Parameter estimation errors

(1) Two waveform models: "true" (GR) and approximate (AP).

(2) The error $\Delta\theta^i = \theta^i_{\rm bf} - \theta^i_{\rm tr}$, in the estimate for parameter θ^i is approximately

$$\Delta\theta^i = [\Gamma^{-1}(\theta_{\rm bf})]^{ij}(\partial_j \mathbf{h}_{\rm AP}(\theta_{\rm bf})|\mathbf{n})$$
$$+[\Gamma^{-1}(\theta_{\rm bf})]^{ij}(\partial_j \mathbf{h}_{\rm AP}(\theta_{\rm bf})|\mathbf{h}_{\rm GR}(\theta_{\rm true}) - \mathbf{h}_{\rm AP}(\theta_{\rm true}))\ ,$$

where $\mathbf{h}$ is the waveform, $\mathbf{n}$ is the noise, $(\ |\ )$ is a noise-weighted inner product, and Γ^{ij} is the Fisher matrix

$$\Gamma^{ij} = \left(\frac{\partial \mathbf{h}}{\partial\theta^i} \,\middle|\, \frac{\partial \mathbf{h}}{\partial\theta^j} \right)\ .$$

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$$\Delta\theta^i = [\Gamma^{-1}(\theta_{\rm bf})]^{ij}(\partial_j \mathbf{h}_{\rm AP}(\theta_{\rm bf})|\mathbf{n}) + [\Gamma^{-1}(\theta_{\rm bf})]^{ij}(\partial_j \mathbf{h}_{\rm AP}(\theta_{\rm bf})|\mathbf{h}_{\rm GR}(\theta_{\rm true}) - \mathbf{h}_{\rm AP}(\theta_{\rm true}))\ ,$$

where $\mathbf{h}$ is the waveform, $\mathbf{n}$ is the noise, $(\ |\ )$ is a noise-weighted inner product, and Γ^{ij} is the Fisher matrix

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statistical error

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where $\mathbf{h}$ is the waveform, $\mathbf{n}$ is the noise, $(\ |\ )$ is a noise-weighted inner product, and Γ^{ij} is the Fisher matrix

$$\Gamma^{ij} = \left(\frac{\partial \mathbf{h}}{\partial \theta^i} \middle| \frac{\partial \mathbf{h}}{\partial \theta^j} \right) .$$

statistical error

systematic error

Parameter estimation errors: progress to date

Barack & Cutler (PRD, 2004) used "analytic kludge" waveforms:

(1) motion: slowly precessing Keplerian orbits

(2) radiation: post-Newtonian and, quadrupole approximation (slow motion)

Barack & Cutler results

S/M^2	0.1	0.1	0.1	0.5	0.5	0.5	1	1	1
$e_{\rm LSO}$	0.1	0.3	0.5	0.1	0.3	0.5	0.1	0.3	0.5
$\Delta(\ln M)$	$2.6e{-4}$	$5.6e{-4}$	$5.3e{-5}$	$2.7e{-4}$	$9.2e{-4}$	$7.7e{-5}$	$2.8e{-4}$	$2.5e{-4}$	$1.5e{-4}$
$\Delta(S/M^2)$	$3.6e{-5}$	$7.9e{-5}$	$4.5e{-5}$	$1.3e{-4}$	$6.3e{-4}$	$5.1e{-5}$	$2.6e{-4}$	$3.7e{-4}$	$2.6e{-4}$
$\Delta(\ln\mu)$	$6.8e{-5}$	$1.5e{-4}$	$7.4e{-5}$	$6.8e{-5}$	$9.2e{-5}$	$1.0e{-4}$	$6.1e{-5}$	$9.1e{-5}$	$1.0e{-3}$
$\Delta(e_0)$	$6.3e{-5}$	$1.3e{-4}$	$2.9e{-5}$	$8.5e{-5}$	$2.8e{-4}$	$3.2e{-5}$	$1.2e{-4}$	$1.1e{-4}$	$1.6e{-4}$
$\Delta(\cos\lambda)$	$6.0e{-3}$	$1.7e{-2}$	$1.3e{-3}$	$1.3e{-3}$	$5.8e{-3}$	$2.4e{-4}$	$6.5e{-4}$	$8.4e{-4}$	$4.7e{-4}$
$\Delta(\Omega_s)$	$1.8e{-3}$	$1.7e{-3}$	$7.9e{-4}$	$2.0e{-3}$	$1.7e{-3}$	$7.6e{-4}$	$2.1e{-3}$	$1.1e{-3}$	$6.7e{-4}$
$\Delta(\Omega_K)$	$5.6e{-2}$	$5.3e{-2}$	$4.7e{-2}$	$5.5e{-2}$	$5.1e{-2}$	$4.7e{-2}$	$5.6e{-2}$	$5.1e{-2}$	$4.8e{-2}$
$\Delta(\tilde{\gamma}_0)$	$4.0e{-1}$	$6.3e{-1}$	$3.8e{-1}$	$1.0e{+0}$	$6.1e{-1}$	$3.9e{-1}$	$9.3e{-1}$	$3.4e{-1}$	$3.9e{-1}$
$\Delta(\Phi_0)$	$2.6e{-1}$	$6.7e{-1}$	$2.2e{-1}$	$1.4e{+0}$	$7.5e{-1}$	$2.7e{-1}$	$1.5e{+0}$	$1.7e{-1}$	$3.3e{-1}$
$\Delta(\alpha_0)$	$6.2e{-1}$	$5.8e{-1}$	$5.5e{-1}$	$6.3e{-1}$	$5.9e{-1}$	$5.6e{-1}$	$6.4e{-1}$	$5.9e{-1}$	$5.9e{-1}$
$\Delta[\ln(\mu/D)]$	$8.7e{-2}$	$3.8e{-2}$	$3.7e{-2}$	$3.8e{-2}$	$3.7e{-2}$	$3.7e{-2}$	$3.8e{-2}$	$7.0e{-2}$	$3.7e{-2}$
$\Delta(t_0)\nu_0$	$4.5e{-2}$	$1.1e{-1}$	$3.3e{-2}$	$2.3e{-1}$	$1.3e{-1}$	$4.4e{-2}$	$2.5e{-1}$	$3.2e{-2}$	$5.5-2$

TABLE III. Parameter extraction accuracy for inspiral of a $10M_\odot$ CO onto a $10^6M_\odot$ MBH at SNR=30 (based on data collected during the last year of inspiral). Shown are results for various values of the MBH's spin magnitude S and the final eccentricity $e_{\rm LSO}$. The rest of the parameters are set as follows: $t_0 = t_{LSO} - (1/2)$yr (middle of integration), $\tilde{\gamma}_0 = 0$, $\Phi_0 = 0$, $\theta_S = \pi/4$, $\phi_S = 0$, $\lambda = \pi/6$, $\alpha_0 = 0$, $\theta_K = \pi/8$, $\phi_K = 0$.

Barack & Cutler results

S/M^2 $e_{\rm LSO}$	0.1 0.1	0.1 0.3	0.1 0.5	0.5 0.1	0.5 0.3	0.5 0.5	1 0.1	1 0.3	1 0.5
$\Delta(\ln M)$	$2.6e{-4}$	$5.6e{-4}$	$5.3e{-5}$	$2.7e{-4}$	$9.2e{-4}$	$7.7e{-5}$	$2.8e{-4}$	$2.5e{-4}$	$1.5e{-4}$
$\Delta(S/M^2)$	$3.6e{-5}$	$7.9e{-5}$	$4.5e{-5}$	$1.3e{-4}$	$6.3e{-4}$	$5.1e{-5}$	$2.6e{-4}$	$3.7e{-4}$	$2.6e{-4}$
$\Delta(\ln \mu)$	$6.8e{-5}$	$1.5e{-4}$	$7.4e{-5}$	$6.8e{-5}$	$9.2e{-5}$	$1.0e{-4}$	$6.1e{-5}$	$9.1e{-5}$	$1.0e{-3}$
$\Delta(e_0)$	$6.3e{-5}$	$1.3e{-4}$	$2.9e{-5}$	$8.5e{-5}$	$2.8e{-4}$	$3.2e{-5}$	$1.2e{-4}$	$1.1e{-4}$	$1.6e{-4}$
$\Delta(\cos\lambda)$	$6.0e{-3}$	$1.7e{-2}$	$1.3e{-3}$	$1.3e{-3}$	$5.8e{-3}$	$2.4e{-4}$	$6.5e{-4}$	$8.4e{-4}$	$4.7e{-4}$
$\Delta(\Omega_s)$	$1.8e{-3}$	$1.7e{-3}$	$7.9e{-4}$	$2.0e{-3}$	$1.7e{-3}$	$7.6e{-4}$	$2.1e{-3}$	$1.1e{-3}$	$6.7e{-4}$
$\Delta(\Omega_K)$	$5.6e{-2}$	$5.3e{-2}$	$4.7e{-2}$	$5.5e{-2}$	$5.1e{-2}$	$4.7e{-2}$	$5.6e{-2}$	$5.1e{-2}$	$4.8e{-2}$
$\Delta(\tilde\gamma_0)$	$4.0e{-1}$	$6.3e{-1}$	$3.8e{-1}$	$1.0e{+0}$	$6.1e{-1}$	$3.9e{-1}$	$9.3e{-1}$	$3.4e{-1}$	$3.9e{-1}$
$\Delta(\Phi_0)$	$2.6e{-1}$	$6.7e{-1}$	$2.2e{-1}$	$1.4e{+0}$	$7.5e{-1}$	$2.7e{-1}$	$1.5e{+0}$	$1.7e{-1}$	$3.3e{-1}$
$\Delta(\alpha_0)$	$6.2e{-1}$	$5.8e{-1}$	$5.5e{-1}$	$6.3e{-1}$	$5.9e{-1}$	$5.6e{-1}$	$6.4e{-1}$	$5.9e{-1}$	$5.9e{-1}$
$\Delta[\ln(\mu/D)]$	$8.7e{-2}$	$3.8e{-2}$	$3.7e{-2}$	$3.8e{-2}$	$3.7e{-2}$	$3.7e{-2}$	$3.8e{-2}$	$7.0e{-2}$	$3.7e{-2}$
$\Delta(t_0)\nu_0$	$4.5e{-2}$	$1.1e{-1}$	$3.3e{-2}$	$2.3e{-1}$	$1.3e{-1}$	$4.4e{-2}$	$2.5e{-1}$	$3.2e{-2}$	$5.5-2$

TABLE III. Parameter extraction accuracy for inspiral of a $10M_\odot$ CO onto a $10^6M_\odot$ MBH at SNR=30 (based on data collected during the last year of inspiral). Shown are results for various values of the MBH's spin magnitude S and the final eccentricity $e_{\rm LSO}$. The rest of the parameters are set as follows: $t_0 = t_{LSO} - (1/2)$yr (middle of integration), $\tilde\gamma_0 = 0$, $\Phi_0 = 0$, $\theta_S = \pi/4$, $\phi_S = 0$, $\lambda = \pi/6$, $\alpha_0 = 0$, $\theta_K = \pi/8$, $\phi_K = 0$.

Some things can be measured "really well",
the rest can be measured, "somewhat well".

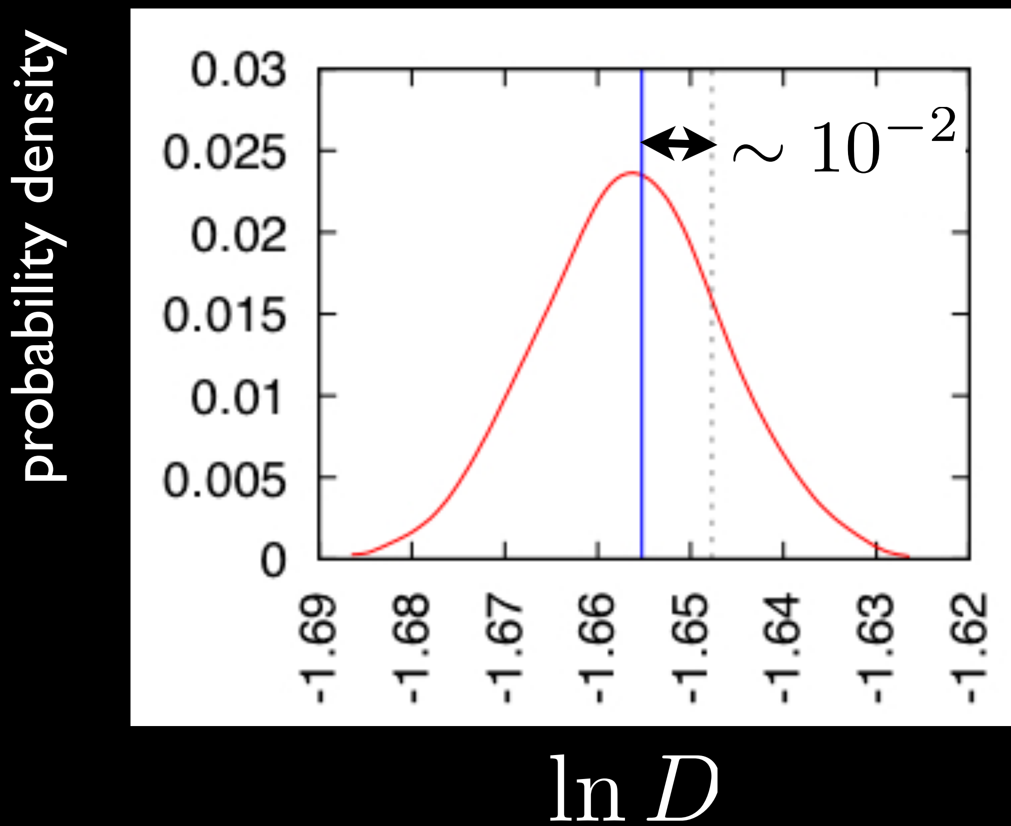
Barack & Cutler results for star in galactic center

S/M^2	0.1	0.1	0.1	0.5	0.5	0.5	1	1	1
e	0.10	0.43	0.80	0.10	0.43	0.80	0.10	0.43	0.80
ν_0(mHz)	0.044	0.035	0.013	0.044	0.035	0.013	0.044	0.035	0.013
SNR	19.8	29.9	26.4	19.0	28.3	25.7	18.8	28.4	29.7
$\Delta(\ln M)$	$3.7e{-3}$	$3.6e{-2}$	$2.1e{-1}$	$5.9e{-3}$	$3.8e{-2}$	$2.1e{-1}$	$1.0e{-2}$	$3.8e{-2}$	$2.1e{-1}$
$\Delta(S/M^2)$	$2.5e{-3}$	$1.9e{-3}$	$6.9e{-3}$	$3.2e{-3}$	$2.5e{-3}$	$5.1e{-3}$	$9.6e{-3}$	$6.8e{-3}$	$9.9e{-3}$
$\Delta(\ln\mu)$	$6.1e{+3}$	$4.0e{+3}$	$4.5e{+3}$	$6.4e{+3}$	$4.3e{+3}$	$4.6e{+3}$	$6.5e{+3}$	$4.2e{+3}$	$4.1e{+3}$
$\Delta(e_0)$	$1.2e{-2}$	$2.4e{-2}$	$3.3e{-2}$	$1.3e{-2}$	$2.5e{-2}$	$3.3e{-2}$	$1.3e{-2}$	$2.5e{-2}$	$3.2e{-2}$
$\Delta(\cos\lambda)$	$3.2e{-2}$	$1.8e{-2}$	$2.0e{-2}$	$2.6e{-2}$	$1.7e{-2}$	$1.8e{-2}$	$2.6e{-2}$	$1.7e{-2}$	$1.6e{-2}$
$\Delta(\Omega_s)$	$3.1e{-2}$	$9.4e{-3}$	$1.1e{-2}$	$2.0e{-2}$	$7.6e{-3}$	$7.0e{-3}$	$2.0e{-2}$	$7.9e{-3}$	$7.1e{-3}$
$\Delta(\Omega_K)$	$3.1e{-2}$	$1.1e{-2}$	$1.0e{-2}$	$1.9e{-2}$	$7.5e{-3}$	$6.2e{-3}$	$2.0e{-2}$	$8.0e{-3}$	$7.5e{-3}$
	$(1.1e{-2})$	$(5.1e{-3})$	$(6.4e{-3})$	$(1.2e{-2})$	$(4.6e{-3})$	$(4.2e{-3})$	$(1.2e{-2})$	$(4.9e{-3})$	$(4.8e{-3})$
$\Delta(\tilde{\gamma}_0)$	$1.6e{+1}$	$3.6e{-1}$	$2.5e{-1}$	$1.5e{+1}$	$3.5e{-1}$	$2.3e{-1}$	$1.3e{+1}$	$3.1e{-1}$	$2.1e{-1}$
$\Delta(\Phi_0)$	$2.0e{+2}$	$3.6e{+0}$	$1.7e{+0}$	$2.0e{+2}$	$3.8e{+0}$	$1.7e{+0}$	$2.1e{+2}$	$3.8e{+0}$	$1.6e{+0}$
$\Delta(\alpha_0)$	$1.8e{-1}$	$7.6e{-2}$	$6.5e{-2}$	$7.4e{-1}$	$7.2e{-2}$	$6.3e{-2}$	$1.5e{+0}$	$7.8e{-2}$	$7.4e{-2}$
$\Delta[\ln(\mu/D)]$	$8.0e{-2}$	$7.8e{-2}$	$2.6e{-1}$	$7.1e{-2}$	$7.8e{-2}$	$2.6e{-1}$	$7.1e{-2}$	$7.7e{-2}$	$2.5e{-1}$
$\Delta(t_0)\nu_0$	$3.1e{+1}$	$6.0e{-1}$	$2.9e{-1}$	$3.2e{+1}$	$6.3e{-1}$	$2.9e{-1}$	$3.2e{+1}$	$6.3e{-1}$	$2.8e{-1}$

TABLE IV. Parameter extraction accuracy for a low-mass main-sequence star at Sgr A*. We assume $M = 2.6\cdot 10^6 M_\odot$, $\mu = 0.06 M_\odot$, and data integration lasting **2 years**. We also assume the star is observed a million years before the (theoretical) plunge, just before tidal effects become important. Each column of the table refers to a different choice of the MBH's spin, orbital eccentricity e and frequency ν at the time of observation. The other parameters are set as follows: $\tilde{\gamma}_0 = 0$, $\Phi_0 = 0$, $\theta_S = 1.66749$ (true value for Sgr A*), $\phi_S = 0$, $\lambda = \pi/6$, $\alpha_0 = 0$, $\theta_K = \pi/8$, $\phi_K = 0$. Most of the values given in the table result from inverting the full, 14×14-d Fisher matrix. The values for $\Delta\Omega_K$ obtained by inverting the 11×11 minor that excludes the CO's mass μ and the two sky-location coordinates θ_S and ϕ_S (whose precise values are known for Sgr A*) are given in parentheses. (For all other parameters, using the known sky position did not signficantly improve measurement accuracy.)

Since then: Mock LISA Data Challenge (MLDC)

The "analytic kludge" waveforms were used in the MLDC (Babak et al, CQG, 2008)

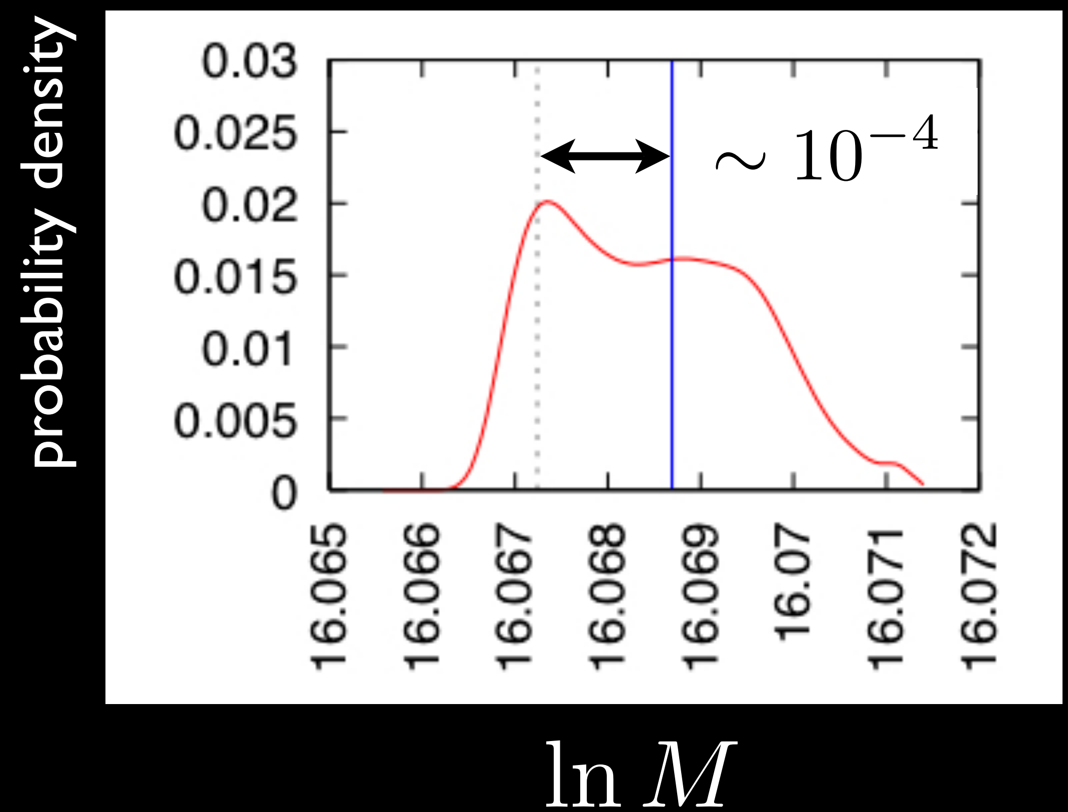


SNR ~ 124
posterior plots
from Cornish, 2009

Results were similar, but a little bit less successful than Barack & Cutler estimates.

Since then: Mock LISA Data Challenge (MLDC)

The "analytic kludge" waveforms were used in the MLDC (Babak et al, CQG, 2008)



SNR ~ 124
posterior plots
from Cornish, 2009

Results were similar, but a little bit less successful than Barack & Cutler estimates.

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(3) Get waves from quadrupole-octupole formula

$$\bar{h}^{jk} = \frac{2}{r}\left(\ddot{I}^{jk} + 2n_i\ddot{S}^{ijk} + n_i\dddot{M}^{ijk}\right)$$

Parameter estimation errors: black-hole geodesics

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Drasco & Cutler: generic black hole orbits

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Huerta & Gair (8 months ago, PRD): restricted to orbits without eccentricity & inclination

Drasco & Cutler: generic black hole orbits

Results: no shocking difference from Barack & Cutler, or from mock LISA data challenge.

Some things can be measured "really well",
the rest can be measured, "somewhat well".

LISA-EMRI results: SNR ~ 140 to 180

Last year of inspiral, at distance of about 1 Gpc, $M = 10^6 M_\odot$, $\mu = 10 M_\odot$

If we average over 4 systems: $a/M = 0.1,\ 0.5$ & $e_{\rm final} = 0.01,\ 0.1$

$\sigma \sim 10^{-2}$	$\sigma \sim 10^{-7}$ to 10^{-5}
$\log D$	$\log M$
r_0/M	$\log \mu$
θ_0/rad	a/M^2
ϕ_0/rad	p_0
$(\theta,\phi)_{\rm sky}/\mathrm{rad}$	e_0
$(\theta,\phi)_{\rm detector}/\mathrm{rad}$	ι_0/rad

LISA-EMRI results: SNR ~ 1500 to 2880

Last year of inspiral, at distance of about 1 Gpc, $M = 10^6 M_\odot$, $\mu = 10^3 M_\odot$

If we average over 4 systems: $a/M = 0.1,\ 0.5$ & $e_{\rm final} = 0.01,\ 0.1$

$\sigma \sim 10^{-3}$	$\sigma \sim 10^{-8}$ to 10^{-5}
$\log D$	$\log M$
r_0/M	$\log \mu$
θ_0/rad	a/M^2
ϕ_0/rad	p_0
$(\theta, \phi)_{\rm sky}/\mathrm{rad}$	e_0
$(\theta, \phi)_{\rm detector}/\mathrm{rad}$	ι_0/rad

summary & future

No “surprises” in improved statistical errors.

Still, a better understanding of errors for these estimates would be useful.

It is time to extend these calculations to LIGO.

It is also time to start exploring theoretical limitations by looking at systematic errors.