Introductory Algebra — Exercise no. 4
Due Thursday, 15 December, 2016

1. Given a matrix representation $\mathcal{D}_R$ of a group $\mathcal{G}$ of elements R. Define the matrices $\{\mathcal{D}_R^*\}$, the complex conjugates of the matrices $\{\mathcal{D}_R\}$, and $\{\widetilde{\mathcal{D}}_R^{-1}\}$, the inverses of the transposes of the matrices $\{\mathcal{D}_R\}$. Show that each of these sets of matrices constitutes a representation of $\mathcal{G}$.

2. Consider the action of $\mathcal{S}_3$, the symmetric group on three objects, on the function of three mutually commuting variables $F(x, y, z) = xy$ and determine the dimension of the invariant vector space that it generates. Construct the matrices of the representation carried by this vector space. Are they unitary?

Find a vector in the space which is invariant under the action of all elements of the group and form an orthonormal basis which includes this vector. [Represent the original vectors which span the space by $(1, 0, \ldots), (0, 1, \ldots)$, etc.] Show that, in this basis, the representation matrices are reduced to a block diagonal form including a one-dimensional representation and a complementary representation. Is the complementary representation reducible?

The multiplication table of $\mathcal{S}_3$, expressed in terms of cycles, is

(1)	(12)	(13)	(23)	(123)	(132)
(12)	(1)	(132)	(123)	(23)	(13)
(13)	(123)	(1)	(132)	(12)	(23)
(23)	(132)	(123)	(1)	(13)	(12)
(123)	(13)	(23)	(12)	(132)	(1)
(132)	(23)	(12)	(13)	(1)	(123)