Introductory Algebra — Exercise no. 3
Due Thursday, 1 December, 2016

1. Given a homomorphism from a group $\mathcal{G}$, of order n, onto a group $\mathcal{G}'$, of order $m < n$. Prove that the elements of $\mathcal{G}$ which are mapped into the identity element $E' \in \mathcal{G}'$ form an invariant subgroup $\mathcal{K} \triangleleft \mathcal{G}$. Show that all the members of each coset of $\mathcal{G}$, relative to $\mathcal{K}$, are mapped into a single element of $\mathcal{G}'$ and that the quotient group $\mathcal{G}/\mathcal{K}$ is isomorphic to $\mathcal{G}'$ under this mapping.

2. Construct the multiplication table of the direct product group $\mathcal{C}_2 \otimes \mathcal{C}_3$ and compare it with those of $\mathcal{C}_6$ and the triangle group. [$\mathcal{C}_r$ is the cyclic group of order r.]

3. Show that any finite group containing, in addition to the identity, only elements of order 2 is Abelian, isomorphic to the multiple direct product $\mathcal{C}_2 \otimes \mathcal{C}_2 \otimes \mathcal{C}_2 \otimes \ldots \otimes \mathcal{C}_2$ and of order a power of 2.