Introductory Algebra — Exercise no. 1
Due Thursday, 17 November, 2016

1. The four symbols $\{A, B, C, D\}$ are arranged on a 4×4 grid as follows

$$
\begin{array}{cccc}
A & B & C & D \\
B & & & \\
C & & & \\
D & & &
\end{array}
$$

Find all ways to fill the empty spaces so that each symbol appears exactly once in each row and column of the grid. Interpreting the results as multiplication tables of groups of four elements, show that there are only two distinct multiplication tables. Show that these tables satisfy the definition of a group.

2. The following are suggested multiplication tables for a group of five elements, constructed as in question 1.

1	A	B	C	D		1	A	B	C	D		1	A	B	C	D
A	1	C	D	B		A	1	C	D	B		A	D	C	1	B
B	C	D	1	A		B	D	1	A	C		B	C	A	D	1
C	D	A	B	1		C	B	D	1	A		C	1	D	B	A
D	B	1	A	C		D	C	A	B	1		D	B	1	A	C

Explain why the first two tables can immediately be disqualified as group multiplication tables, and confirm that the third table is the table of a group.